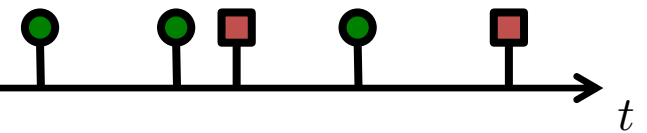


# Learning with Temporal Point Processes



## Models & Inference

Isabel Valera

Max Planck Institute for Intelligent Systems

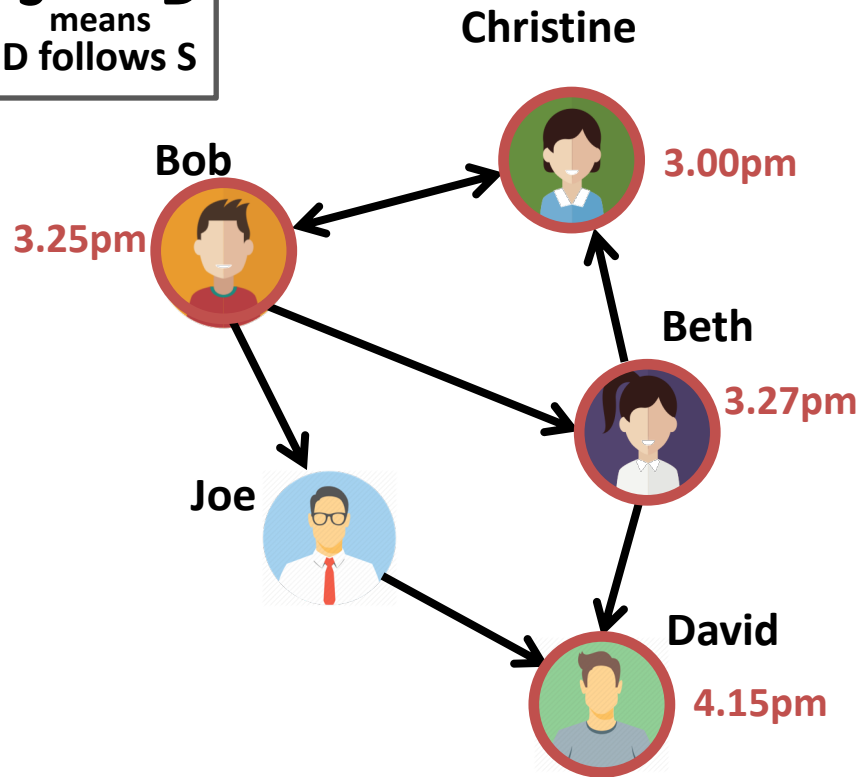
ICML TUTORIAL, JULY 2018

# Models & Inference

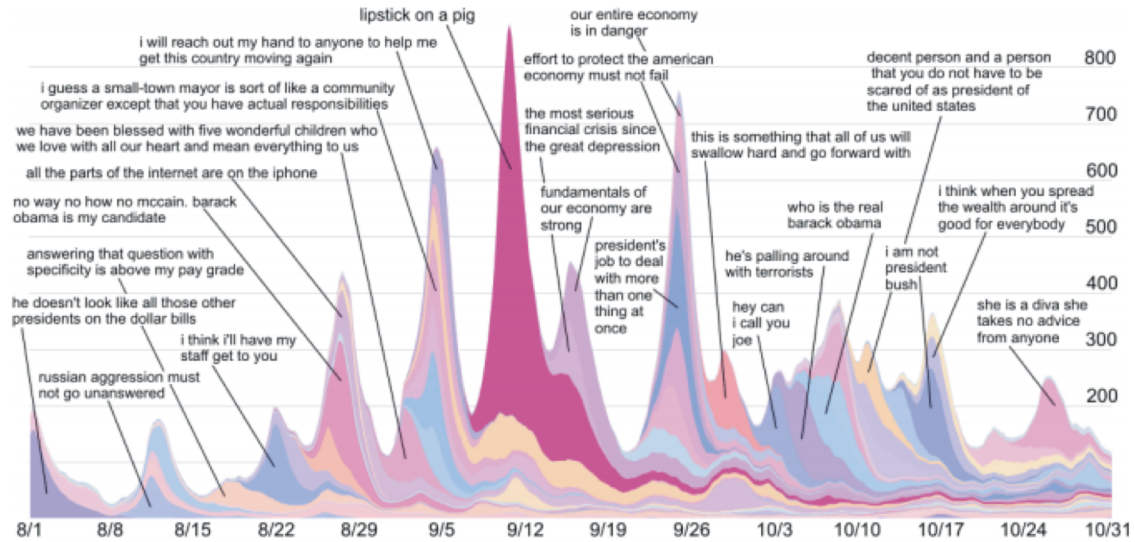
- 1. Modeling event sequences**
2. Clustering event sequences
3. Capturing complex dynamics
4. Causal reasoning on event sequences

# Event sequences as cascades

$S \rightarrow D$   
S means  
D follows S

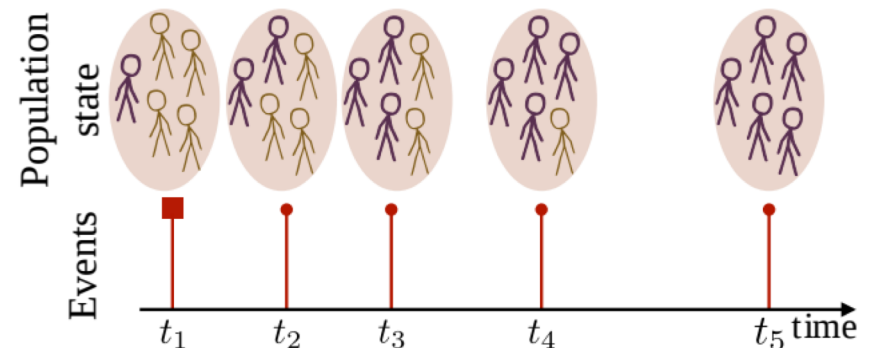
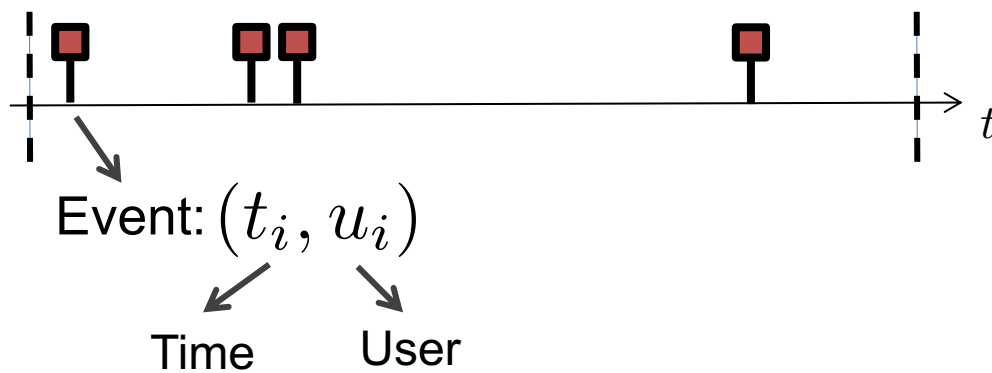


## Information Diffusion



[Leskovec et al., 2009]

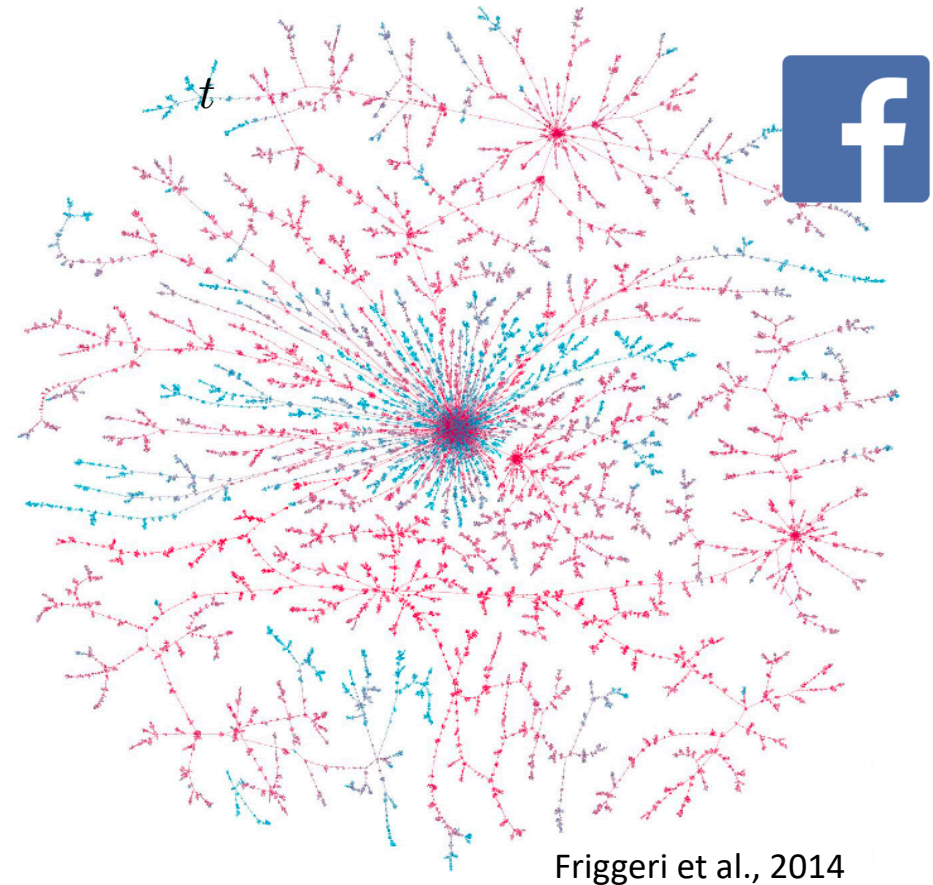
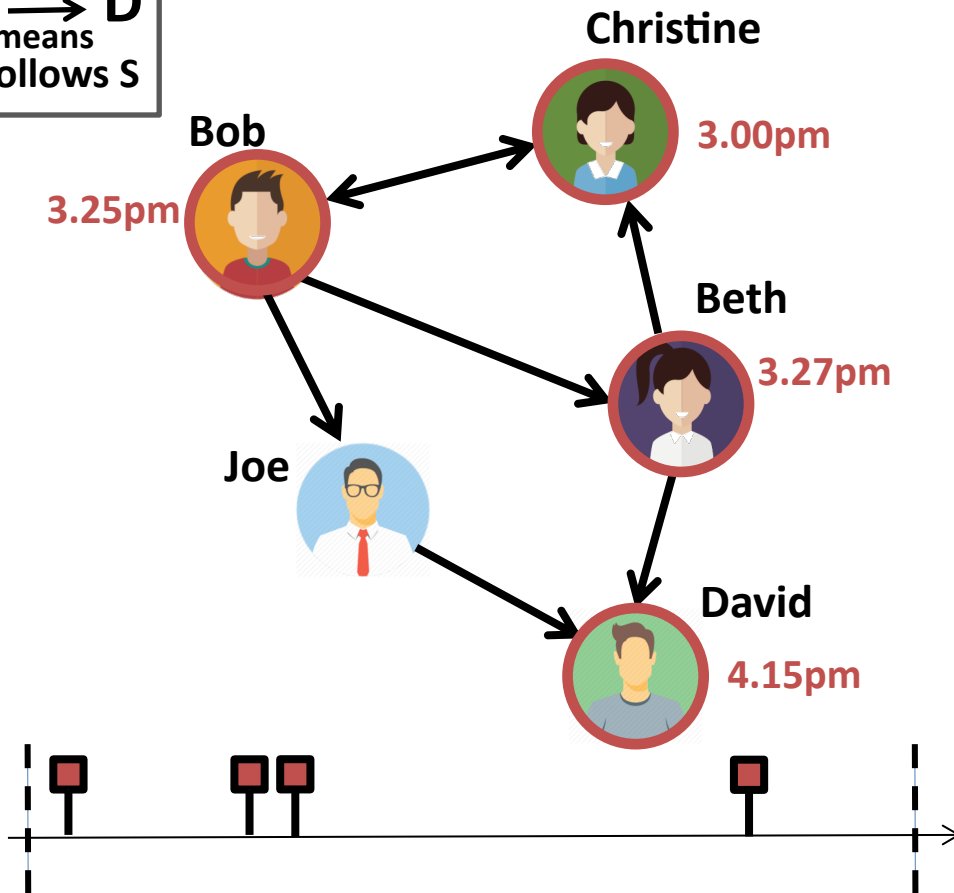
## Disease Diffusion



[Rizoiu et al., 2018]

# An example: idea adoption

$S \rightarrow D$   
means  
D follows S



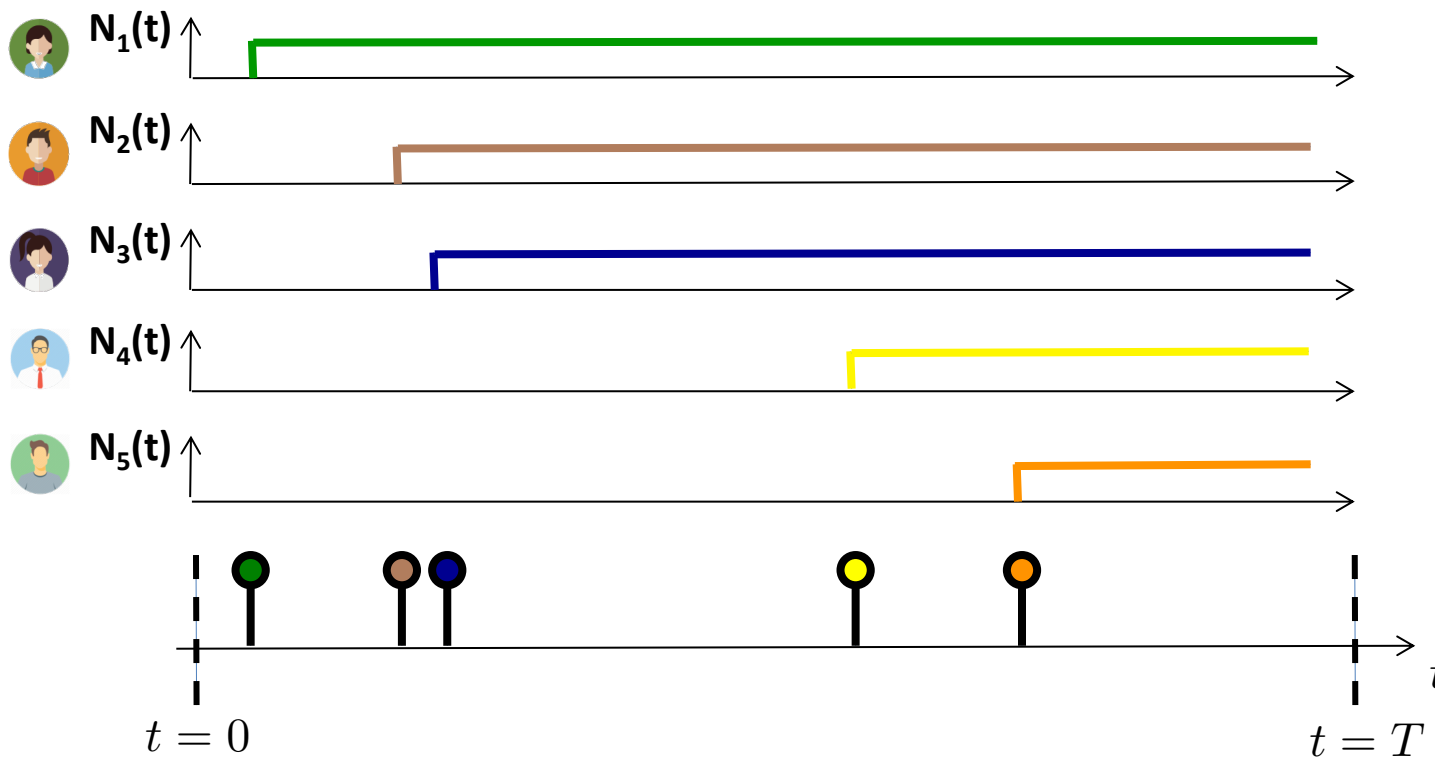
They can have an impact  
in the off-line world

**theguardian**

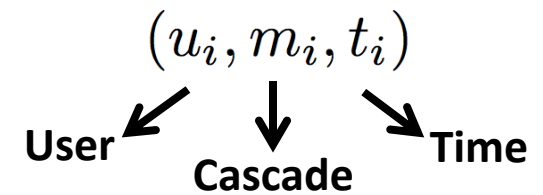
Click and elect: how fake news helped Donald Trump win a real election

# Infection cascade representation

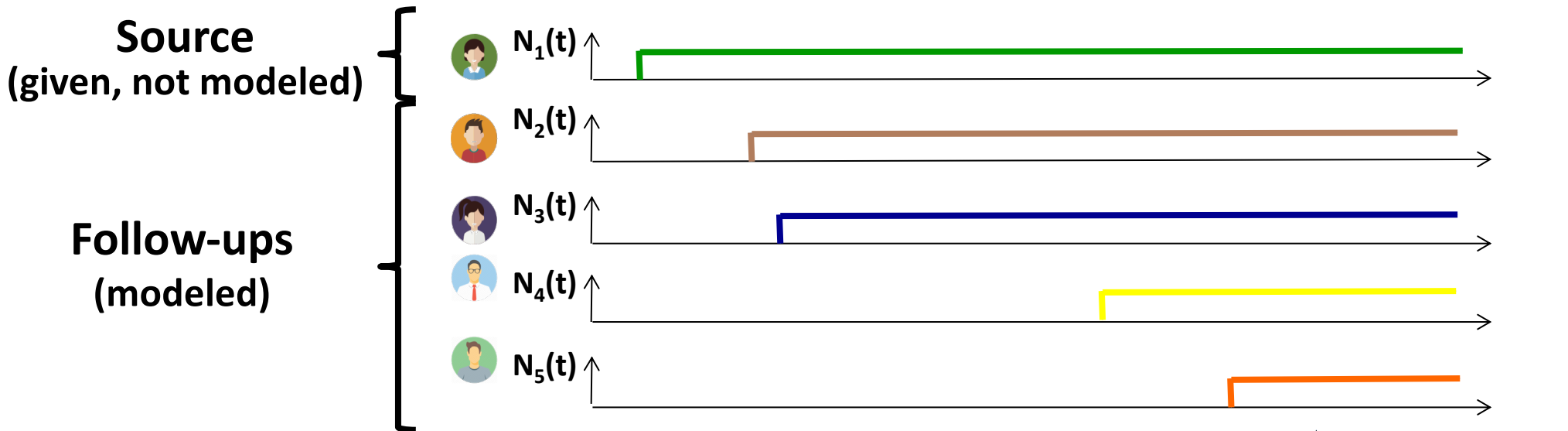
We represent an infection cascade using **terminating temporal point processes**:



**Infection event:**



# Infection intensity

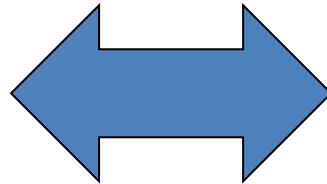


$$\lambda_u^*(t) = \underbrace{(1 - N_u(t))}_{\text{Users get infected only once}} \sum_{v \in [m]} \underbrace{b_{vu} \sum_{e_i \in \mathcal{H}_v(t)} \kappa(t - t_i)}_{\text{Influence from user } v \text{ on user } u \text{ Previous infections of user } v}$$

# Model inference from multiple cascades

Conditional intensities

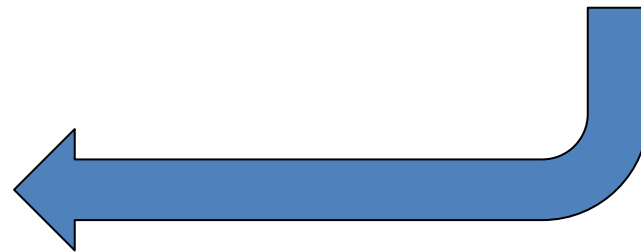
$$\lambda_u^*(t)$$



Diffusion log-likelihood

$$\mathcal{L} = \sum_{u=1}^n \log \lambda_u^*(t_u) - \int_0^T \lambda_u^*(\tau) d\tau$$

Maximum likelihood approach to find model parameters!



Sum up log-likelihoods of multiple cascades!

**Theorem.** For any choice of parametric memory, the **maximum likelihood** problem is **convex in B**.

# Example of real-world diffusion process

Youtube video: <http://youtu.be/hBeaSfRCU4c>

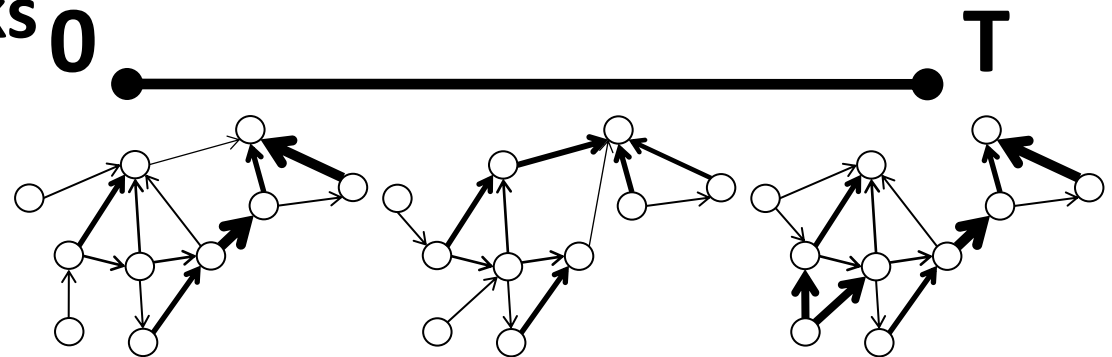


# Dynamic influence

In some cases, influence change over time:



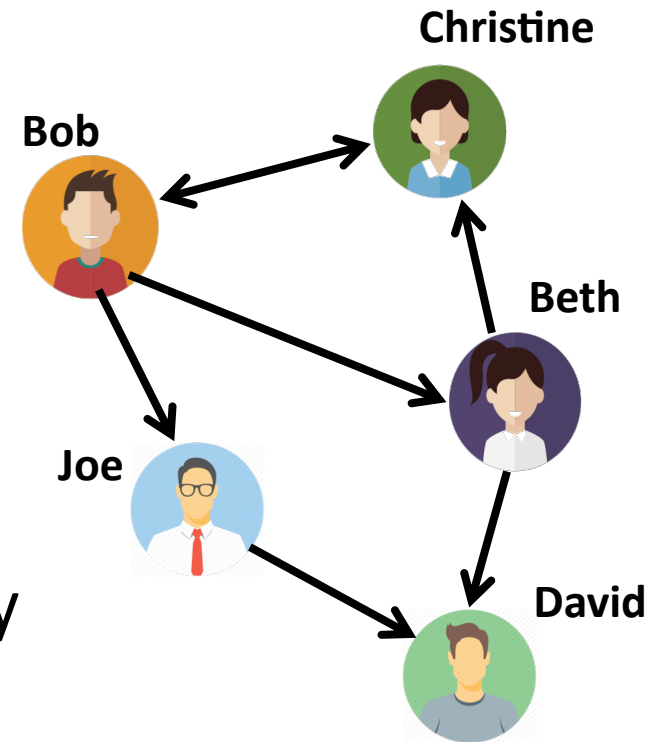
Propagation over networks  
with variable influence



# Recurrent events: beyond cascades

**Up to this point**, each user is only infected once, and event sequences can be seen as cascades.

**In general**, users perform recurrent events over time. E.g., people repeatedly express their opinion online:



How social media is revolutionizing debates

*The New York Times*

*Social Media Are Giving a Voice to Taste Buds*



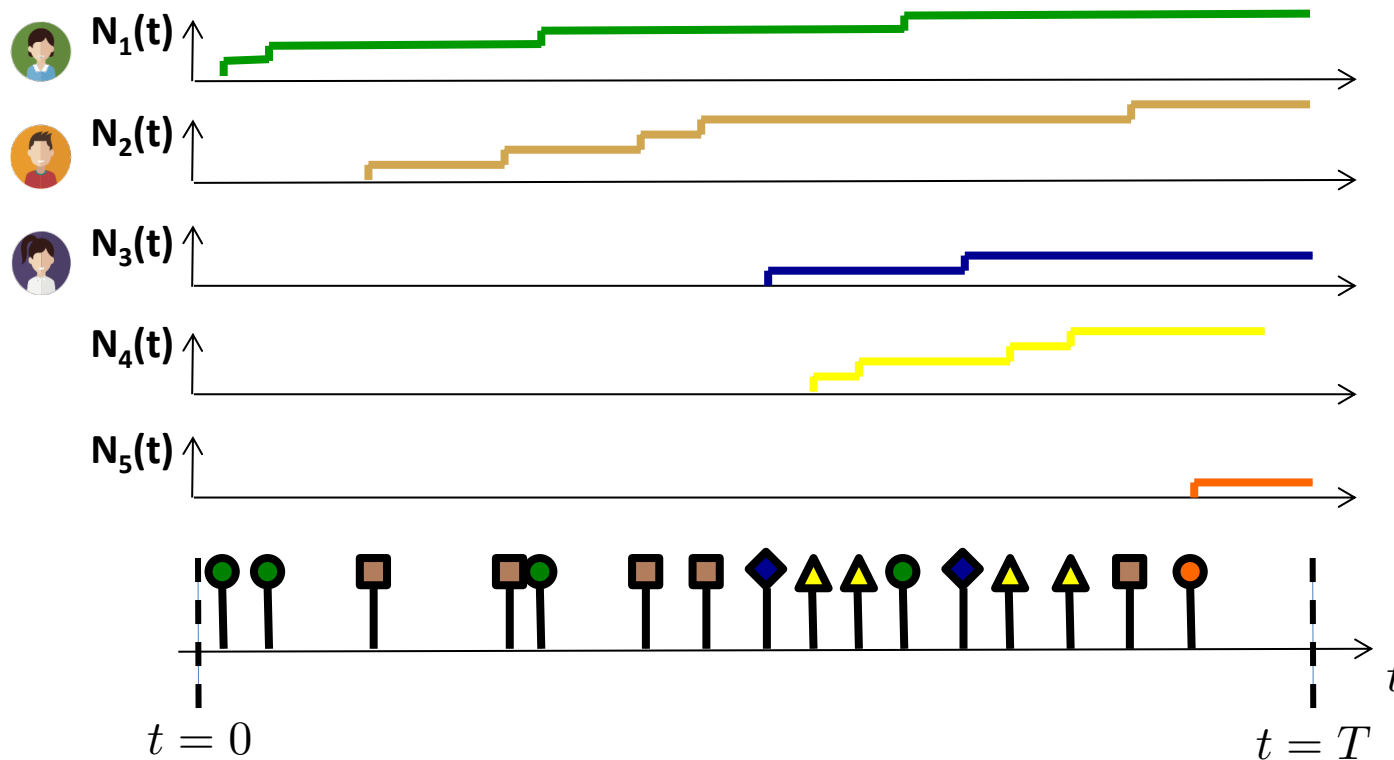
**Twitter Unveils A New Set Of Brand-Centric Analytics**

*The New York Times*

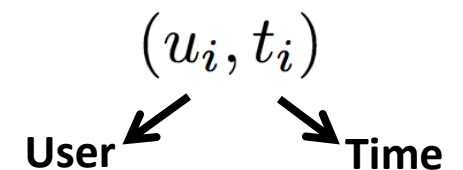
*Campaigns Use Social Media to Lure Younger Voters*

# Recurrent events representation

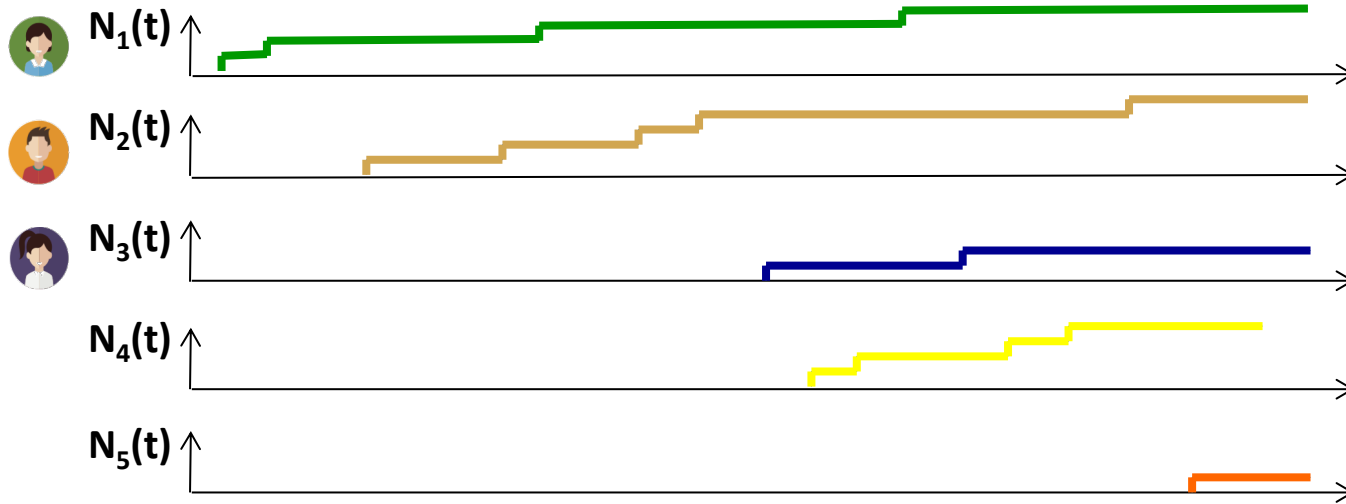
We represent messages using **nonterminating temporal point processes**:



**Recurrent event:**

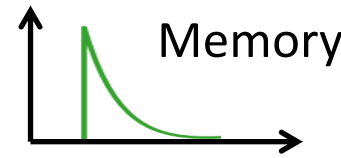


# Recurrent events intensity



Cascade sources!

$$\underbrace{\lambda_u^*(t)}_{\text{User's intensity}} = \underbrace{\mu_u}_{\text{Events on her own initiative}} + \sum_{v \in [m]} \underbrace{b_{vu}}_{\text{Influence from user } v \text{ on user } u} \underbrace{\sum_{e_i \in \mathcal{H}_v(t)} \kappa(t - t_i)}_{\text{Previous messages by user } v}$$



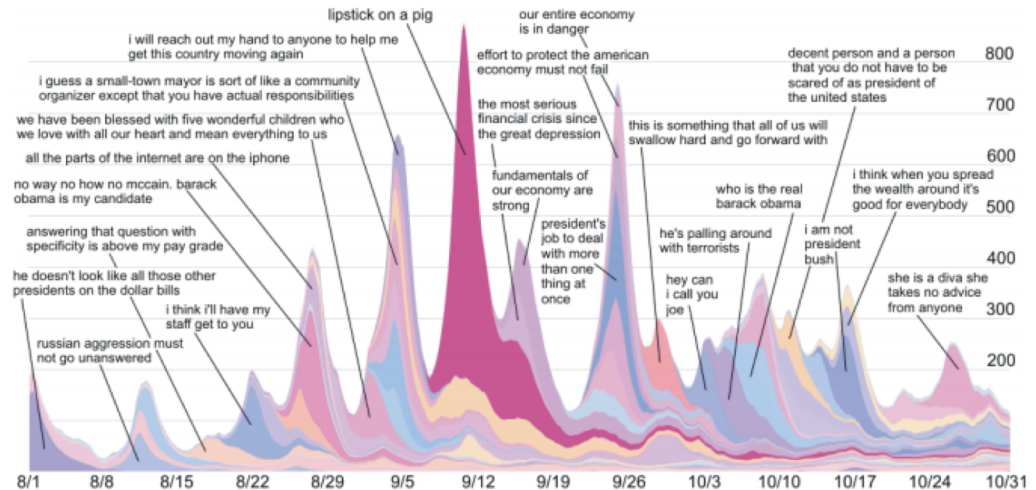
Hawkes process

# Models & Inference

1. Modeling event sequences
- 2. Clustering event sequences**
3. Capturing complex dynamics
4. Causal reasoning on event sequences

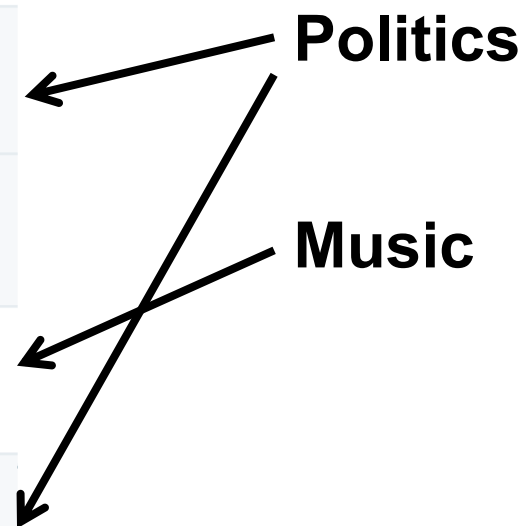
# Event sequences

So far, we have assumed the cascade (topic, meme, etc.) that each event belongs to was known.



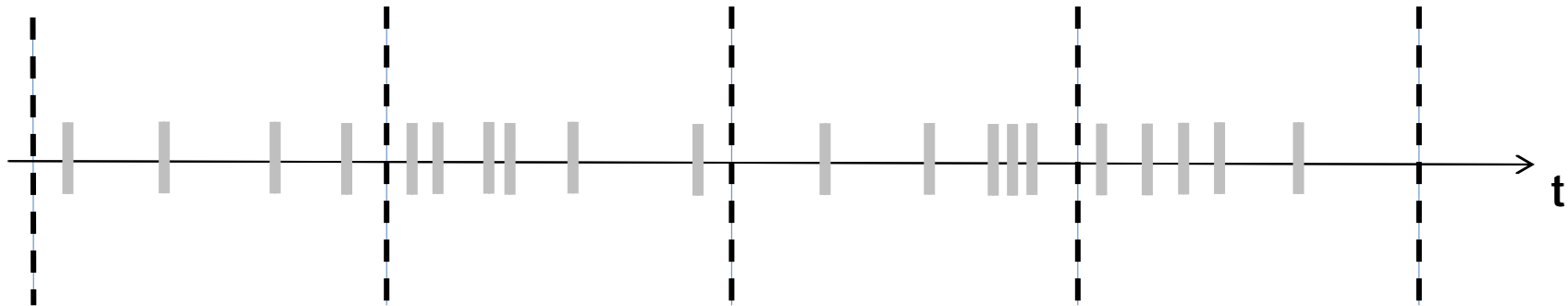
Often, the cluster (topic, meme, etc.) that each event in a sequence belongs to is not known:

-  **BBC News (World)** @BBCWorld · 4m  
Turkey election: Erdogan win ushers in new presidential era
-  **BBC News (World)** @BBCWorld · 46m  
Dublin church: Seven injured as car hits pedestrians
-  **BBC News (World)** @BBCWorld · 2h  
Nigerian music star D'banj's son 'drowns at home'
-  **BBC News (World)** @BBCWorld · 2h  
Turkey election: Country's heart split over Erdogan victory

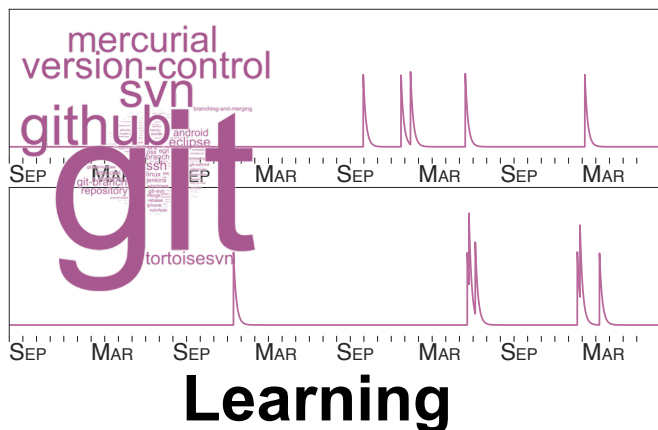


# Clustering event sequences

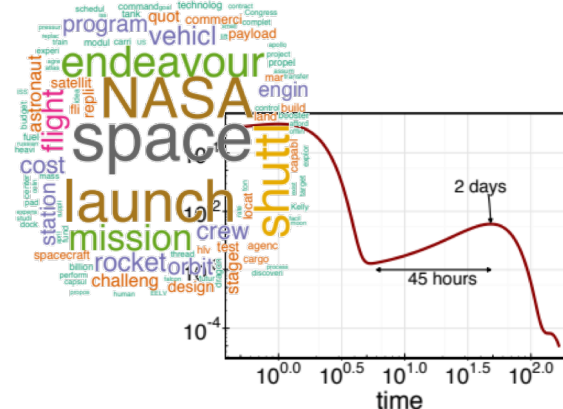
Assume the event **cluster** to be hidden and aim to automatically **learn the cluster assignments** from the data:



**Bayesian methods** to cluster event sequences in the context of:



## Online News



## Health care

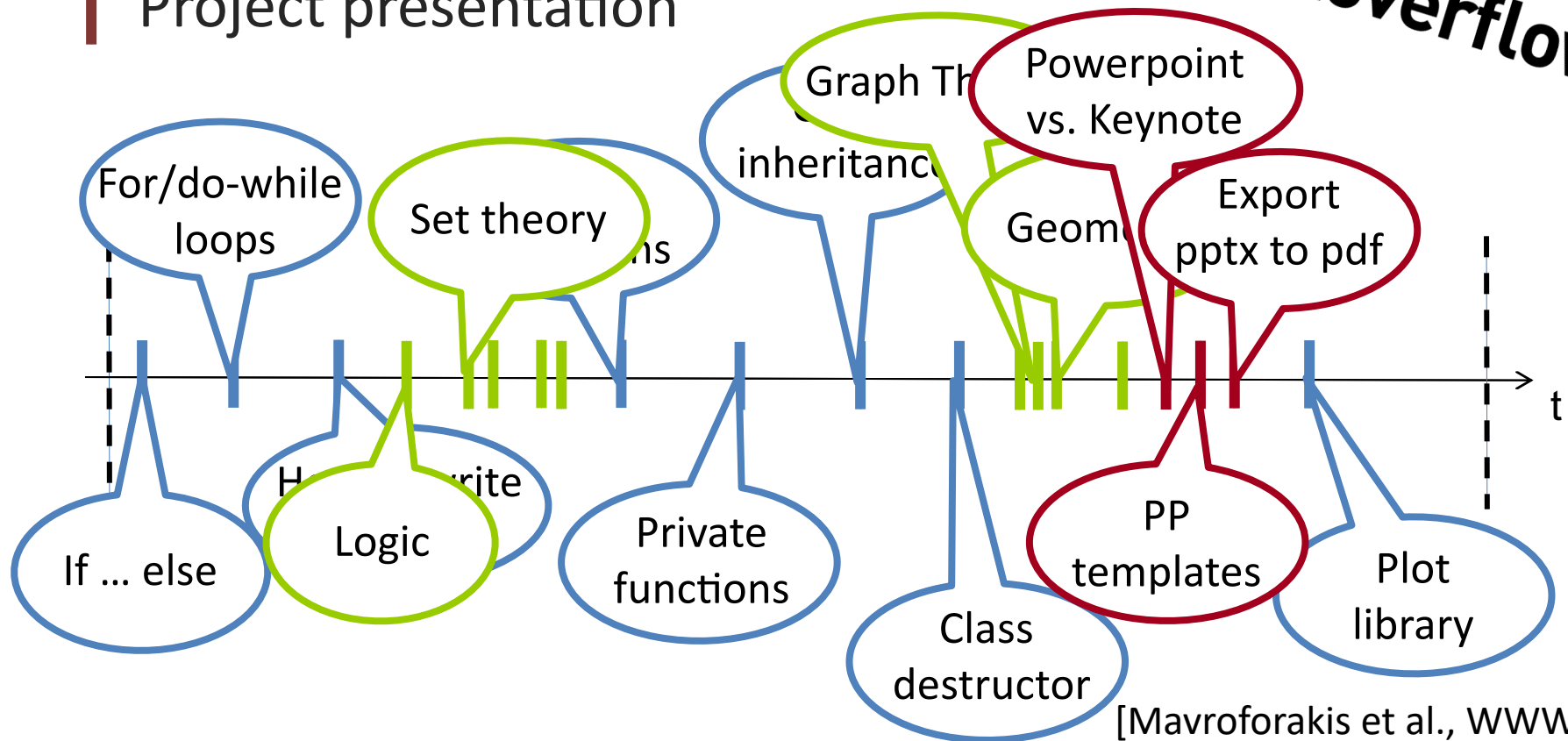
Method	DMHP
ICU Patient	<b>0.3778</b>
IPTV User	<b>0.2004</b>

# Hierarchical Dirichlet Hawkes process



## 1st year computer science student

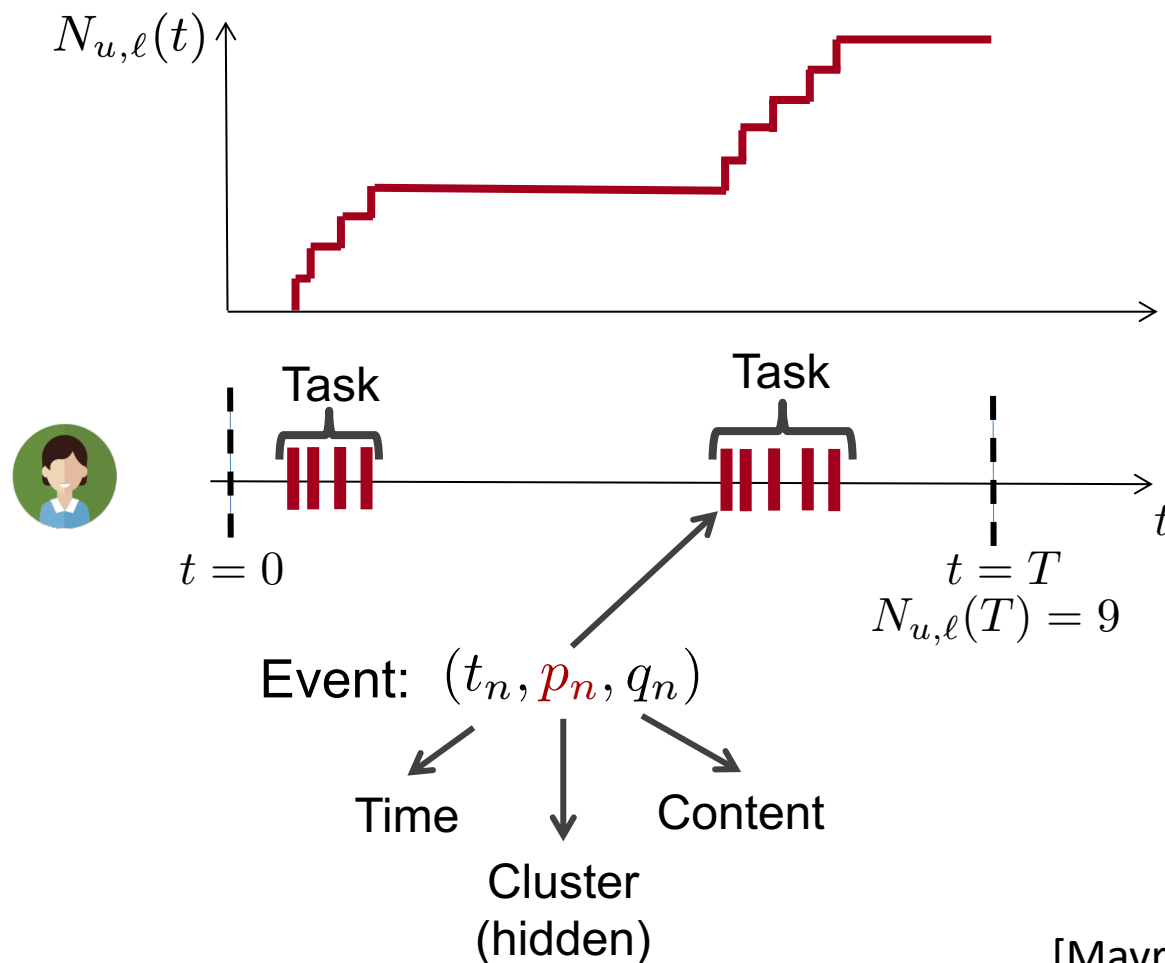
- Introduction to programming
- Discrete math
- Project presentation



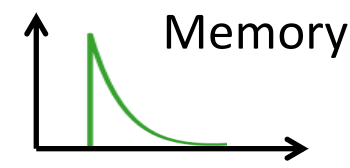
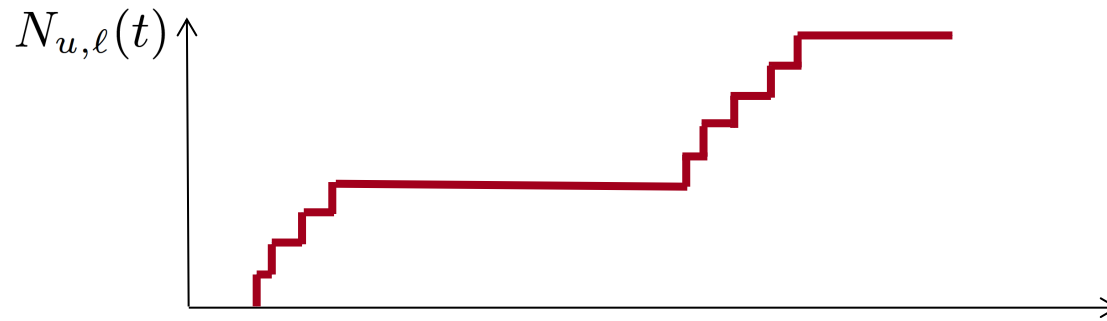


# Events representation

We represent the events using **marked temporal point processes**:



# Cluster intensity



New cascade rate

Cluster popularity

$$\underbrace{\lambda_{u,l}^*(t)}_{\text{Intensity or rate (events / hour)}} = \underbrace{\mu_u \pi_l}_{\text{Own initiative}} + \underbrace{\sum_{j:t_j \in \mathcal{H}_{u,l}(t)} k_{\theta_l}(t - t_j)}_{\text{Follow-up}}$$

Intensity or rate (events / hour)

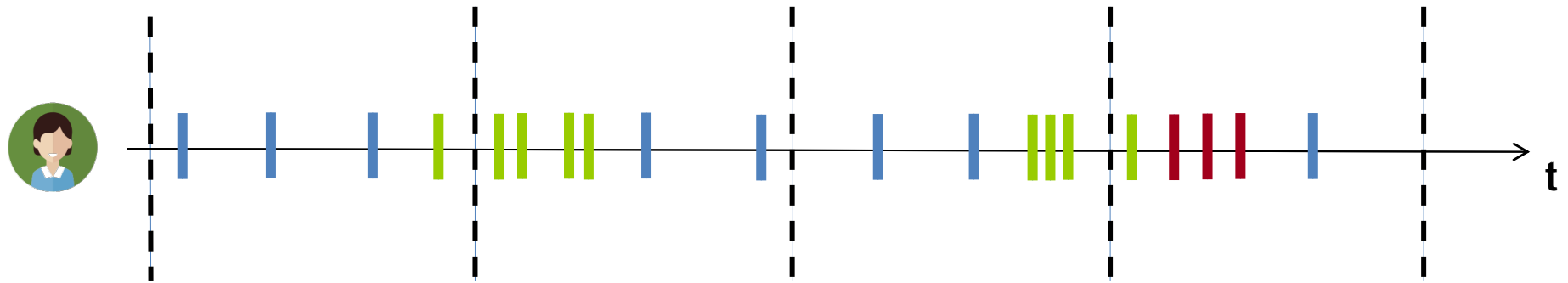
Own initiative

Follow-up

Hawkes process

# User events intensity

Users adopt more than one cluster:



A user's learning events as a multidimensional Hawkes:

$$\begin{array}{l} \text{Time} \searrow \\ \text{cluster} \swarrow \\ (t_n, p_n) \sim \text{Hawkes} \end{array} \begin{pmatrix} \lambda_{u,1}^*(t) \\ \vdots \\ \lambda_{u,\infty}^*(t) \end{pmatrix}$$

$$\text{Content} \rightarrow q_n = \omega \quad \omega_j \sim \text{Multinomial}(\theta_p)$$

# People share same clusters

*Different users adopt same clusters*



Efficient model inference using  
Sequential Montecarlo!

Clus

- Shared parameters across users.

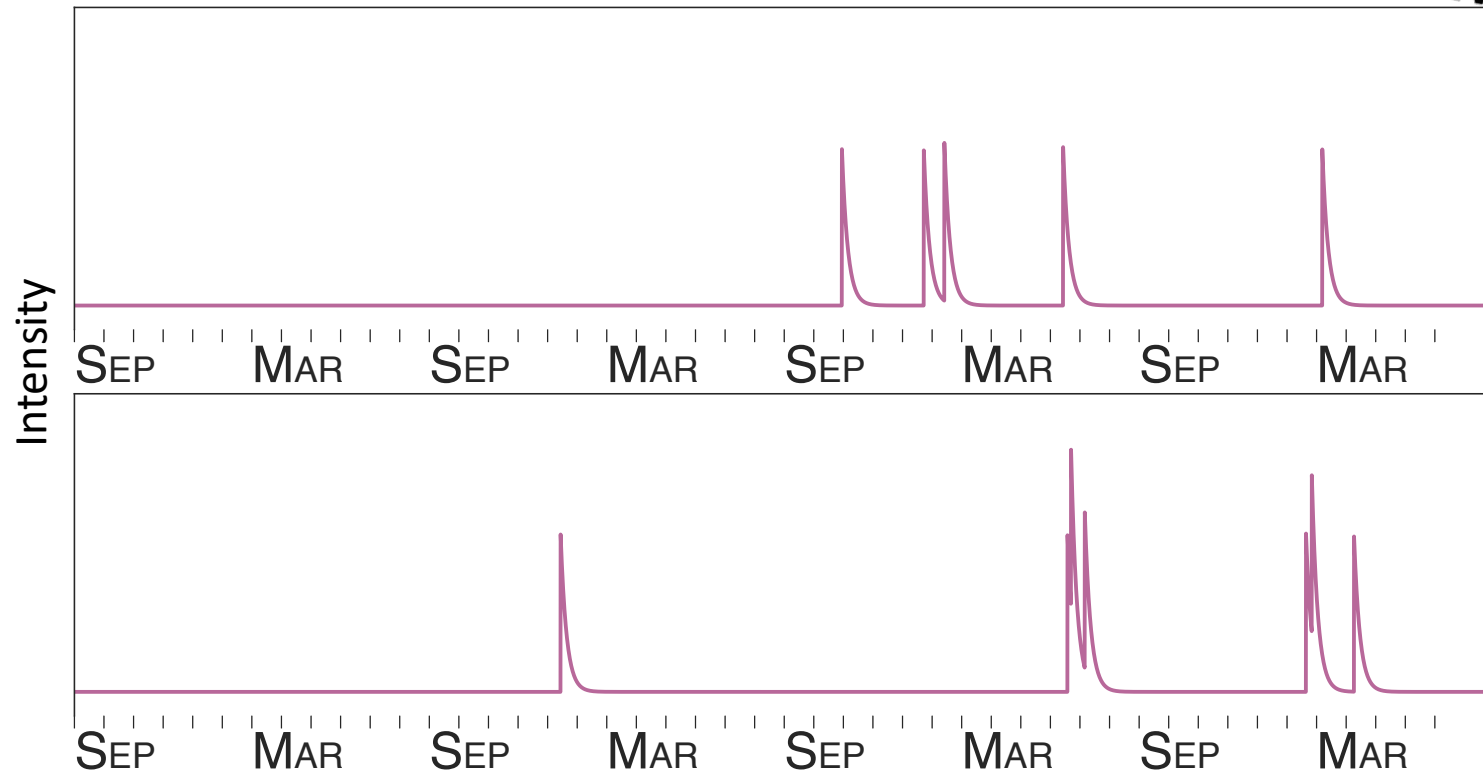
*Details in the  
reference below!*

# Learning cluster (I): Version Control

Content



Intensities



**Version control tasks tend to be specific,  
quickly solved after performing few questions**



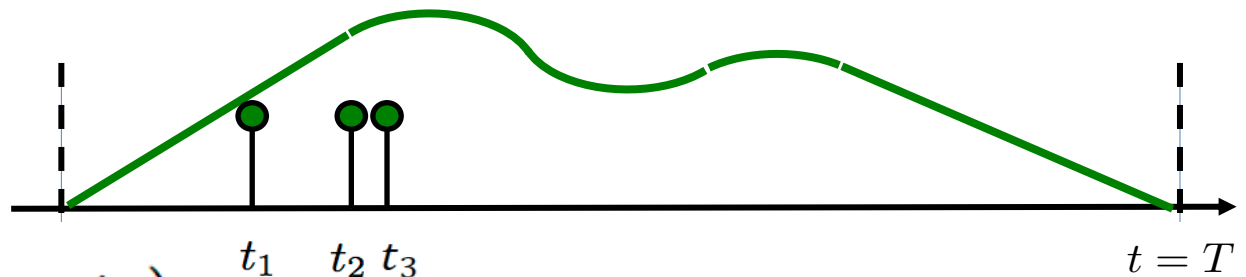
# Models & Inference

1. Modeling event sequences
2. Clustering event sequences
- 3. Capturing complex dynamics**
4. Causal reasoning on event sequences

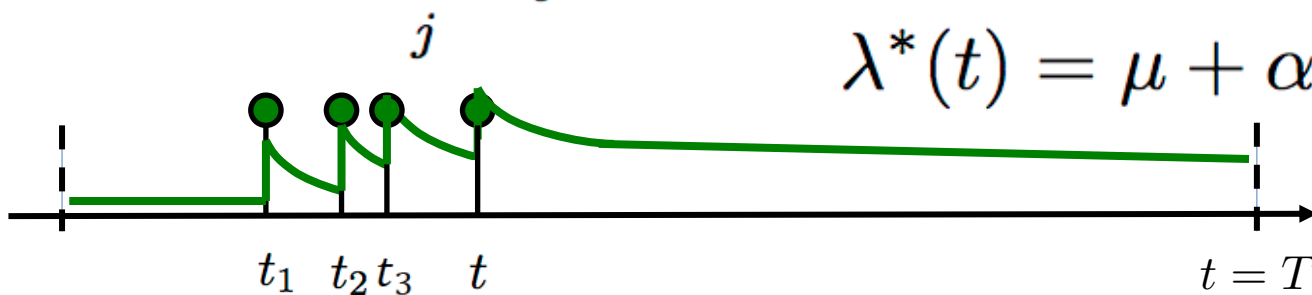
# Towards real-world temporal dynamics

Up to now, we have focused on simple temporal dynamics (and intensity functions):

$$\lambda^*(t) = \mu$$



$$\lambda^*(t) = \sum_j \alpha_j k(t - t_j)$$



$$\lambda^*(t) = \mu + \alpha \sum_{t_i \in \mathcal{H}(t)} \kappa_\omega(t - t_i)$$

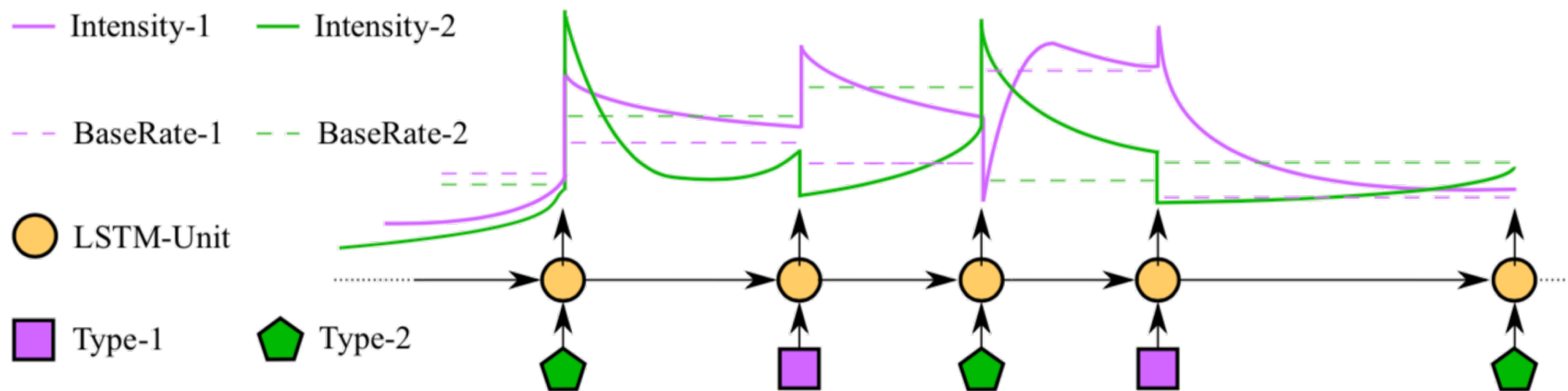
Recent works make use of **RNNs** to capture more complex dynamics

[Du et al., 2016; Dai et al., 2016; Mei & Eisner, 2017; Jing & Smola, 2017; Trivedi et al., 2017; Xiao et al., 2017a; 2018]



# Neural Hawkes process

- 1) History effect does not need to be additive
- 2) Allows for complex memory effects (such as delays)



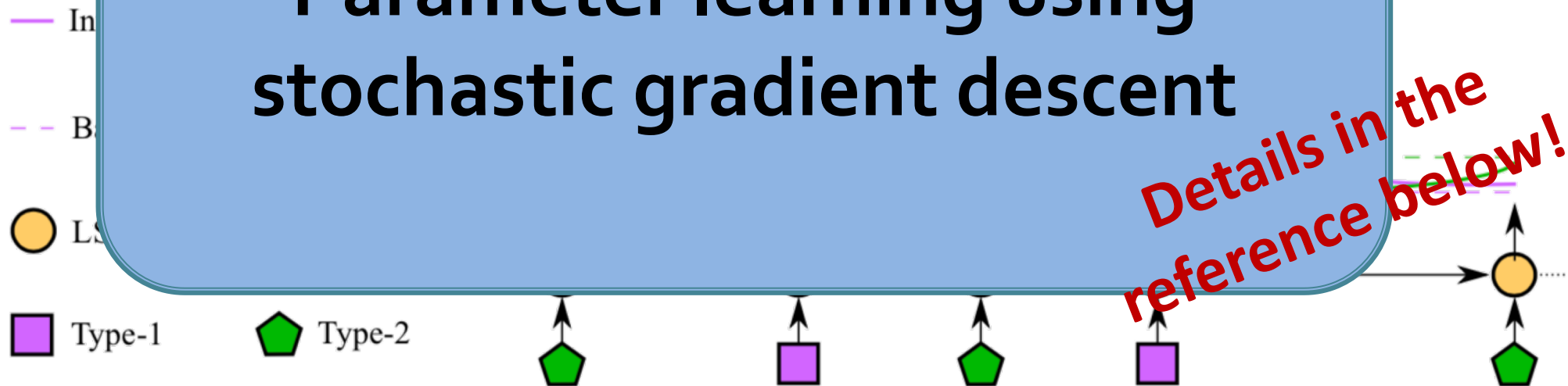
# Neural Hawkes process

$$\lambda_u(t) = f_u(\mathbf{w}_u^\top \mathbf{h}(t))$$

$$\mathbf{h}(t) = \text{DNN}(\mathcal{U}(t))$$

Memory

Parameter learning using  
stochastic gradient descent

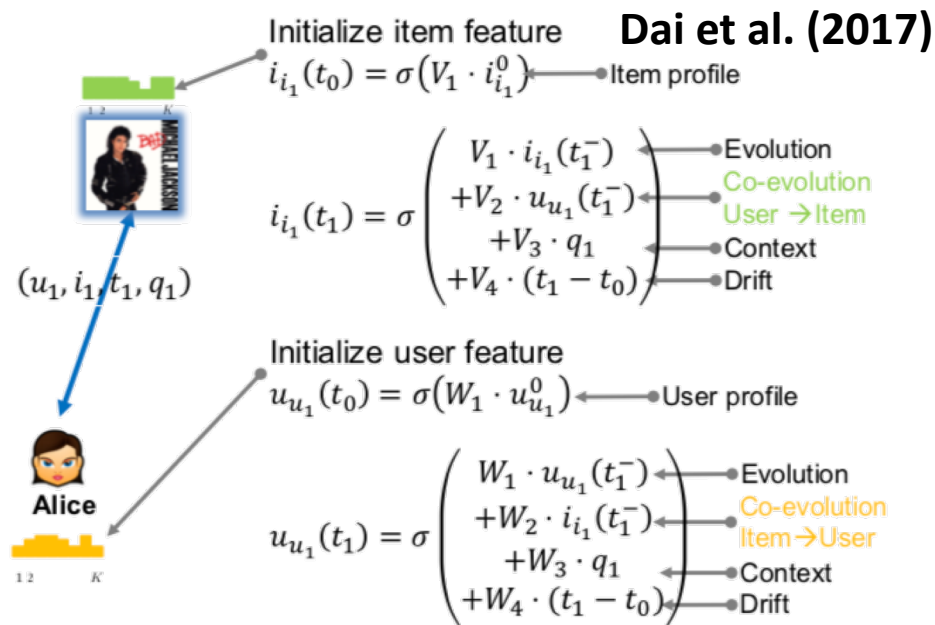


# Applications (I): Predictive Models

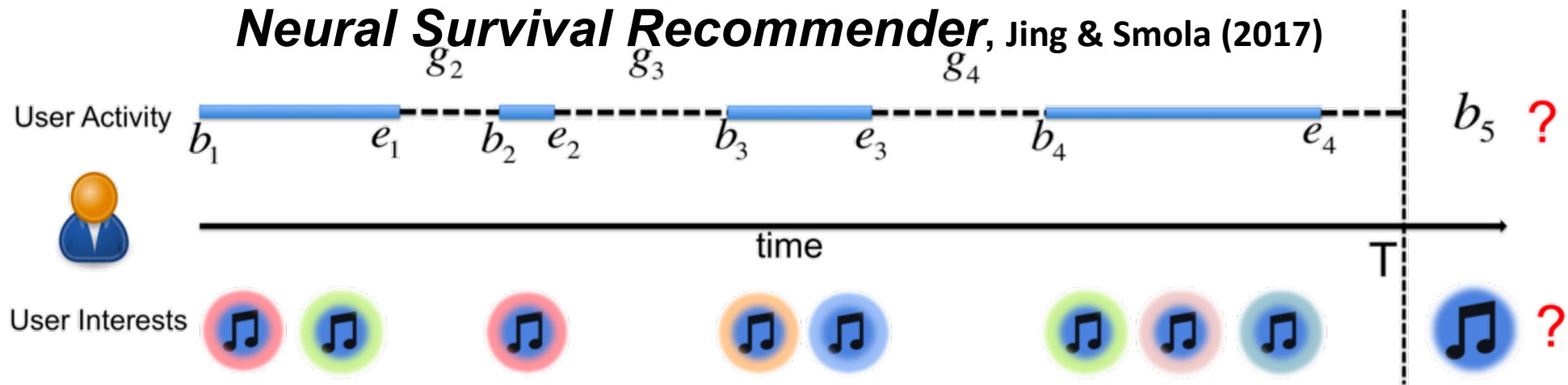
## Know-Evolve, Trivedi et al. (2017)



## Coevolutionary Embedding, Dai et al. (2017)

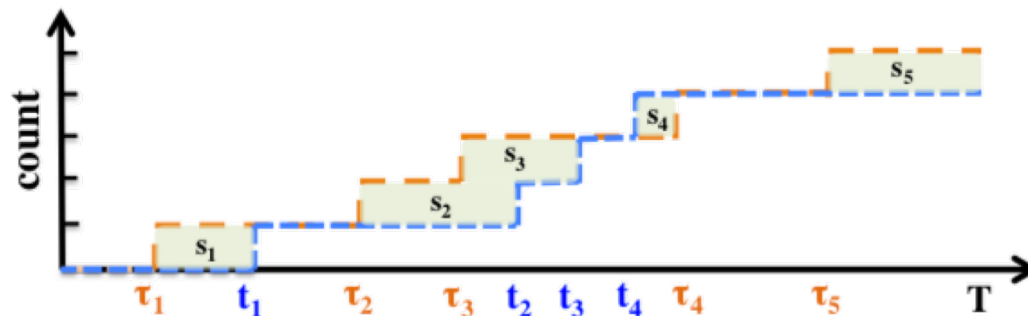


## Neural Survival Recommender, Jing & Smola (2017)

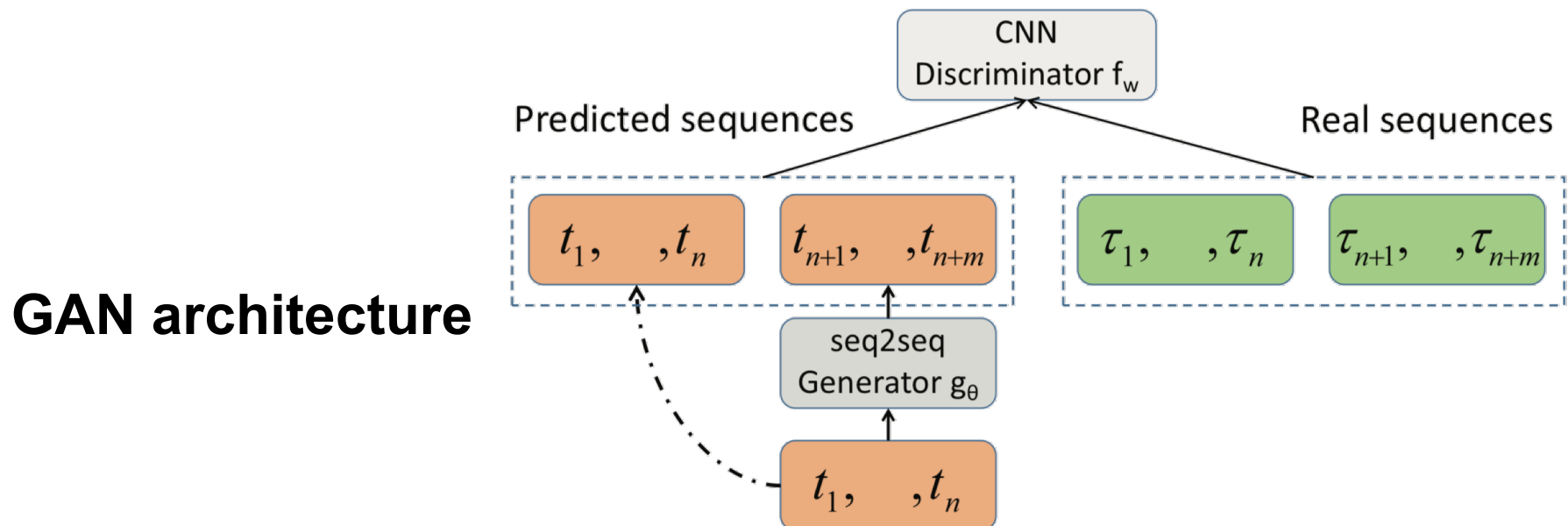


# Applications (II): Generative Models

**Key idea:** Intensity- and likelihood-free models



**Wasserstein-Distance for Temporal Point Processes**



# Models & Inference

1. Modeling event sequences
2. Clustering event sequences
3. Capturing complex dynamics
- 4. Causal reasoning on event sequences**

# Temporal point processes beyond prediction

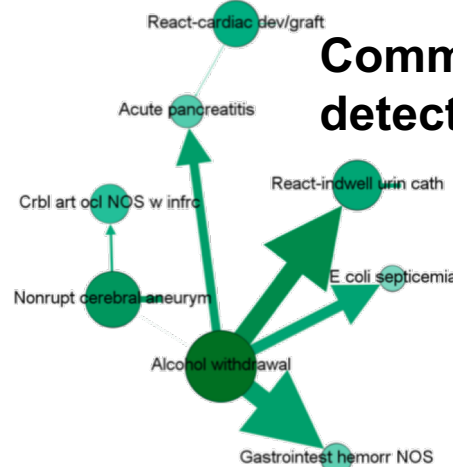
So far, we have focused on models that improve predictions:

Link prediction



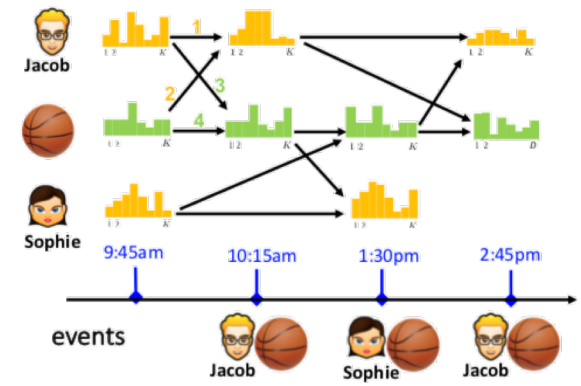
[Trivedi et al., 2017]

Community detection



[Xiao et al., 2017]

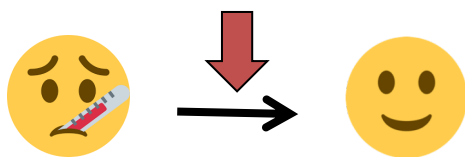
Recommendations



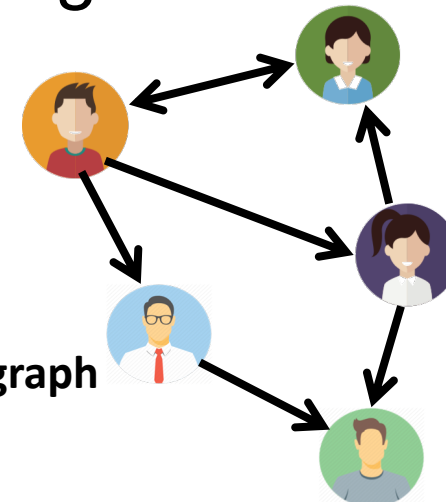
[Dai et al., 2017]

Recent works have focused on performing **causal inference using event sequences**:

Treatment effect



Granger causality graph



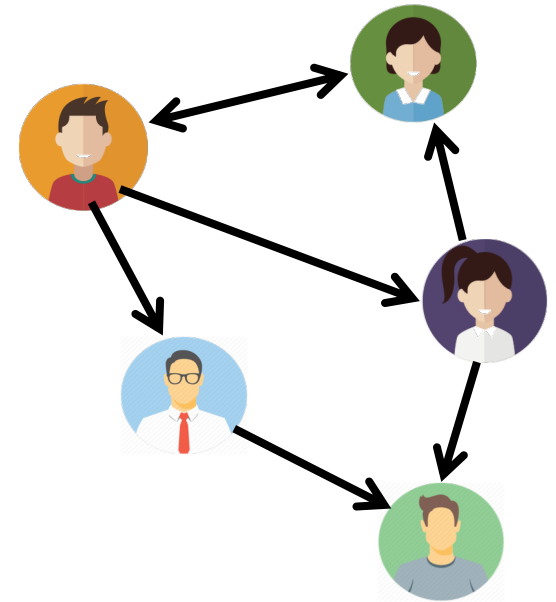
[Xu et al., 2016; Achab et al., 2017; Kuśmierczyk & Gomez-Rodriguez, 2018]

# Uncovering Causality from Hawkes Processes

## Multivariate Hawkes process:

$$N(t) = \sum_{u \in \mathcal{U}} N_u(t)$$

$$\lambda_u(t) = \mu_u + \sum_{v \in \mathcal{U}} \underbrace{\int_0^t k_{u,v}(t-t') dN_v(t')}_{\text{Effect of } v\text{'s past events on } u}$$



## Granger causality:

"X causes Y in the sense of Granger causality if forecasting future values of Y is more successful while taking X past values into account"

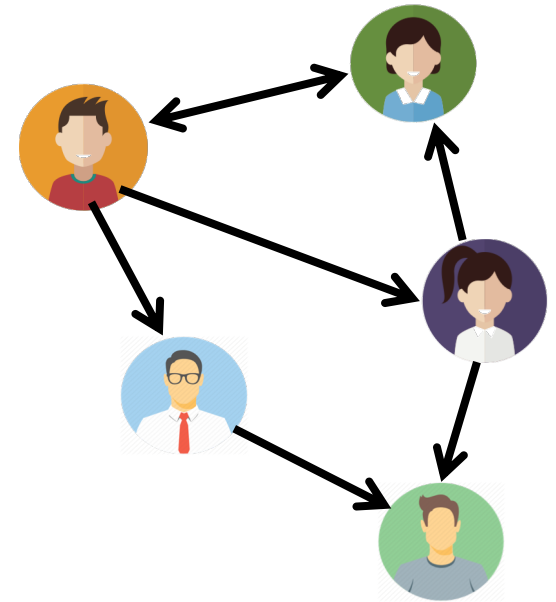
[Granger, 1969]

# Uncovering Causality from Hawkes Processes

**Multivariate Hawkes process:**

$$N(t) = \sum_{u \in \mathcal{U}} N_u(t)$$

$$\lambda_u(t) = \mu_u + \sum_{v \in \mathcal{U}} \underbrace{\int_0^t k_{u,v}(t-t') dN_v(t')}_{\text{Effect of } v\text{'s past events on } u}$$



**Granger causality on multivariate Hawkes processes:**

“  $N_v(t)$  does not Granger-cause  $N_u(t)$  w.r.t.  $N(t)$  if and only if  $k_{u,v}(\tau) = 0$  for  $\tau \in \mathbb{R}^+$  ”

[Eichler et al., 2016]

[Achab et al., ICML 2017]

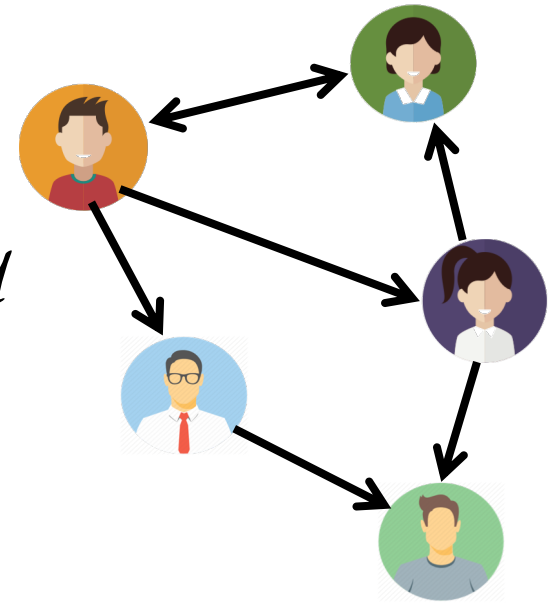


# Uncovering Causality from Hawkes Processes

Goal is to estimate  $G = [g_{uv}]$ , where:

$$g_{uv} = \int_0^{+\infty} k_{u,v}(\tau) d\tau \geq 0 \text{ for all } u, v \in \mathcal{U}$$

Average total # of events of node  $u$  whose *direct* ancestor is an event by node  $v$



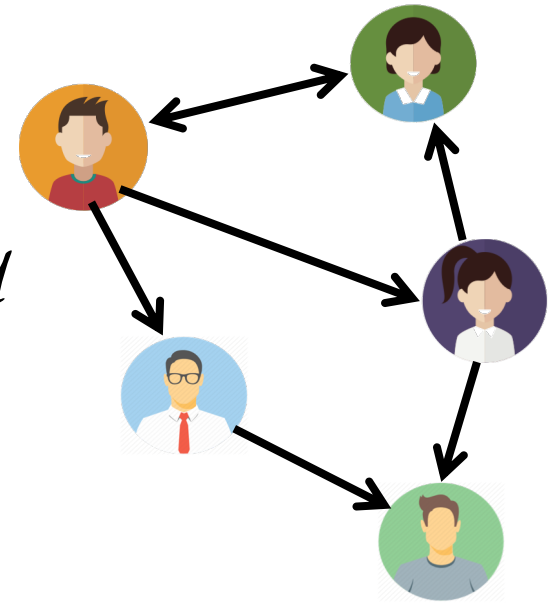
Then,  $G = [g_{uv}]$  quantifies the *direct causal relationship* between nodes.

# Uncovering Causality from Hawkes Processes

Goal is to estimate  $G = [g_{uv}]$ , where:

$$g_{uv} = \int_0^{+\infty} k_{u,v}(\tau) d\tau \geq 0 \text{ for all } u, v \in \mathcal{U}$$

Average total # of events of node  $u$  whose *direct* ancestor is an event by node  $v$



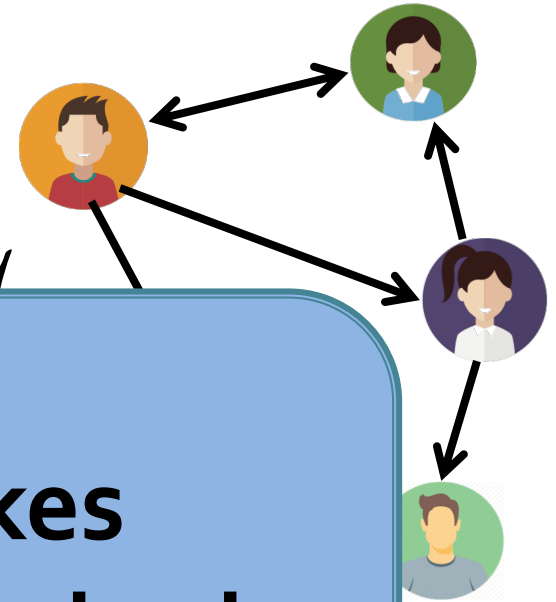
Then,  $G = [g_{uv}]$  quantifies the *direct causal relationship* between nodes.

**Key idea:** Estimate  $G$  using the cumulants  $dN(t)$  of the Hawkes process.

# Uncovering Causality from Hawkes Processes

Goal is to estimate  $G = [g_{uv}]$ , where:

$$g_{uv} = \int_0^{+\infty} k_{uv}(\tau) d\tau > 0 \text{ for all } u, v \in \mathcal{U}$$



**Non parametric Hawkes  
cumulant estimation method**

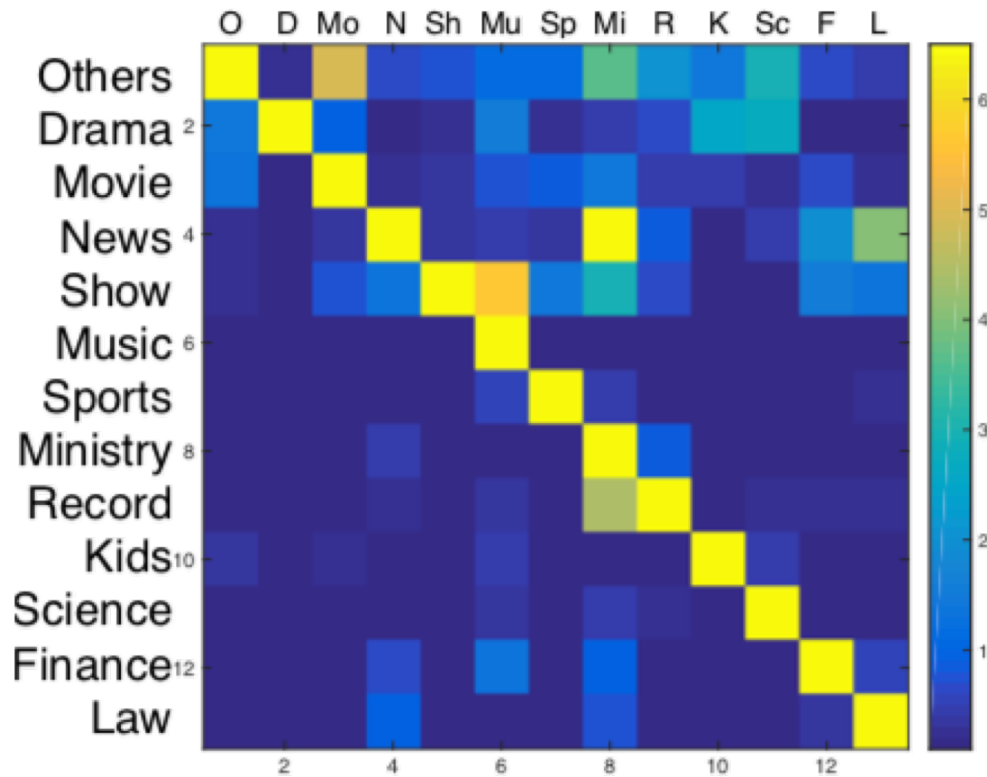
(with TensorFlow implementation)

*Details in the **hip**  
reference below!*

**Key idea:** Estimate  $G$  using the cumulants the  $dN(t)$  of the Hawkes process.

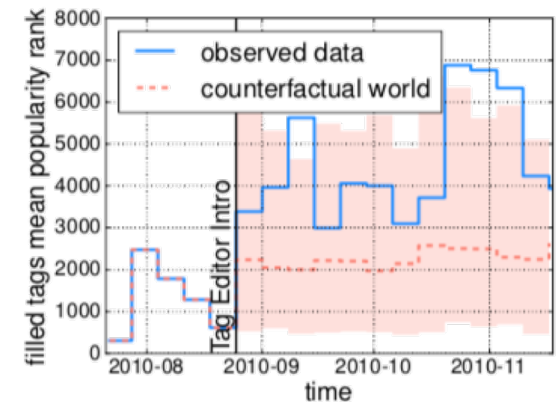
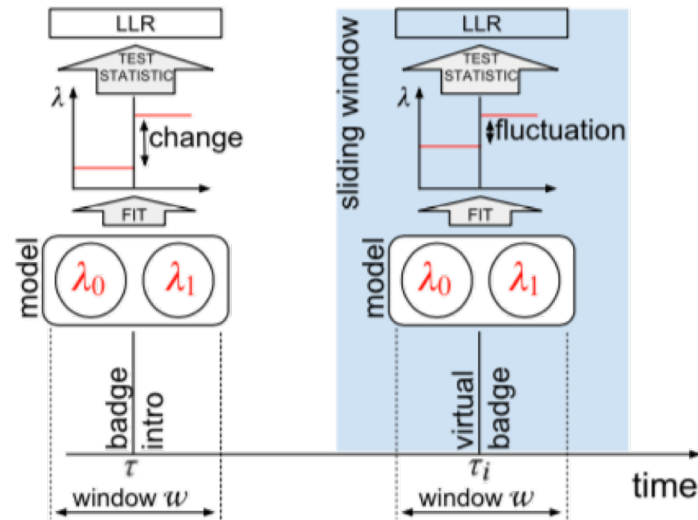
# Causal reasoning: Applications

## Infectivity matrix estimation



[Xu et al., 2016]

## Effect of Badges



Tag wiki rank over time

[Kuśmierczyk & Gomez-Rodriguez, 2018]

# Outline of the Seminar

## TEMPORAL POINT PROCESSES (TPPs): INTRO

1. Intensity function
2. Basic building blocks
3. Superposition
4. Marks and SDEs with jumps

## MODELS & INFERENCE

1. Modeling event sequences
2. Clustering event sequences
3. Capturing complex dynamics
4. Causal reasoning on event sequences

**This  
lecture**

## RL & CONTROL

1. Marked TPPs: a new setting
2. Stochastic optimal control
3. Reinforcement learning

**Next  
lecture**

Slides/references: [learning.mpi-sws.org/tpp-icml18](http://learning.mpi-sws.org/tpp-icml18)