

# **Machine learning for** **Dynamic Social Network Analysis**

## **Applications: Control**

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# Outline of the Seminar

## **REPRESENTATION: TEMPORAL POINT PROCESSES**

1. Intensity function
2. Basic building blocks
3. Superposition
4. Marks and SDEs with jumps

## **APPLICATIONS: MODELS**

1. Information propagation
2. Information reliability
3. Knowledge acquisition

## **APPLICATIONS: CONTROL**

1. Activity shaping
2. When-to-post

**Next**

# Applications: Control

1. Activity shaping
2. When-to-post

# Activity shaping

Can we **steer users' activity** in a **social network in general?**

Why this goal?



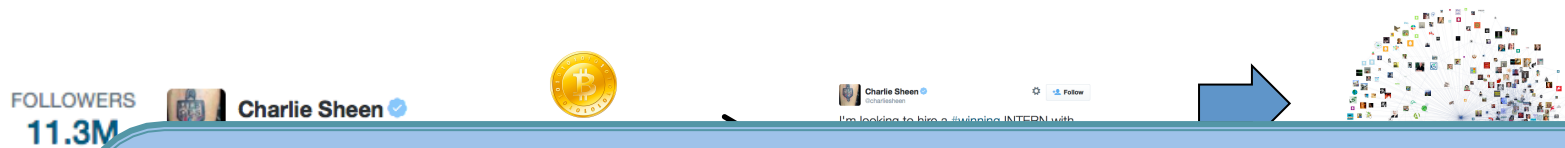
**Twitter Stock Tumbles After Drop in User Engagement**



**7 Ways to Increase Your Social Media Engagement**

# Activity shaping vs influence maximization

## Related to Influence Maximization Problem



**Activity shaping is a generalization  
of influence maximization**

**Influence  
Maximization**

**fixed  
incentive**

**same piece of  
information**

**maximizing  
adoption**

**Activity  
Shaping**

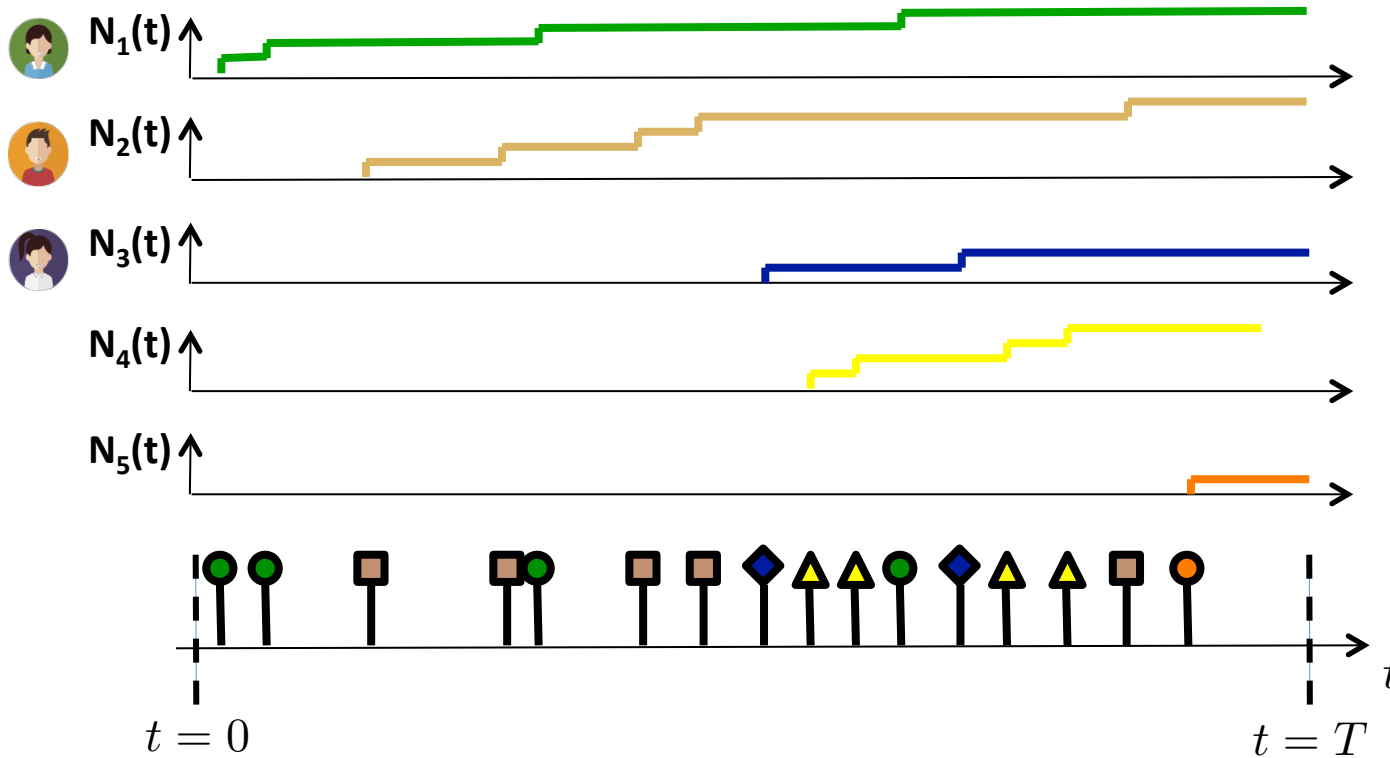
**Variable  
incentive**

**Multiple times  
multiple pieces,  
recurrent!**

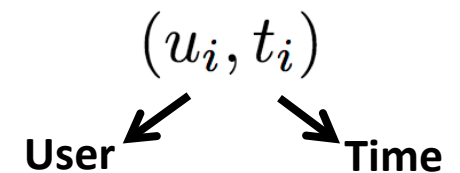
**Many different  
activity shaping  
tasks**

# Event representation

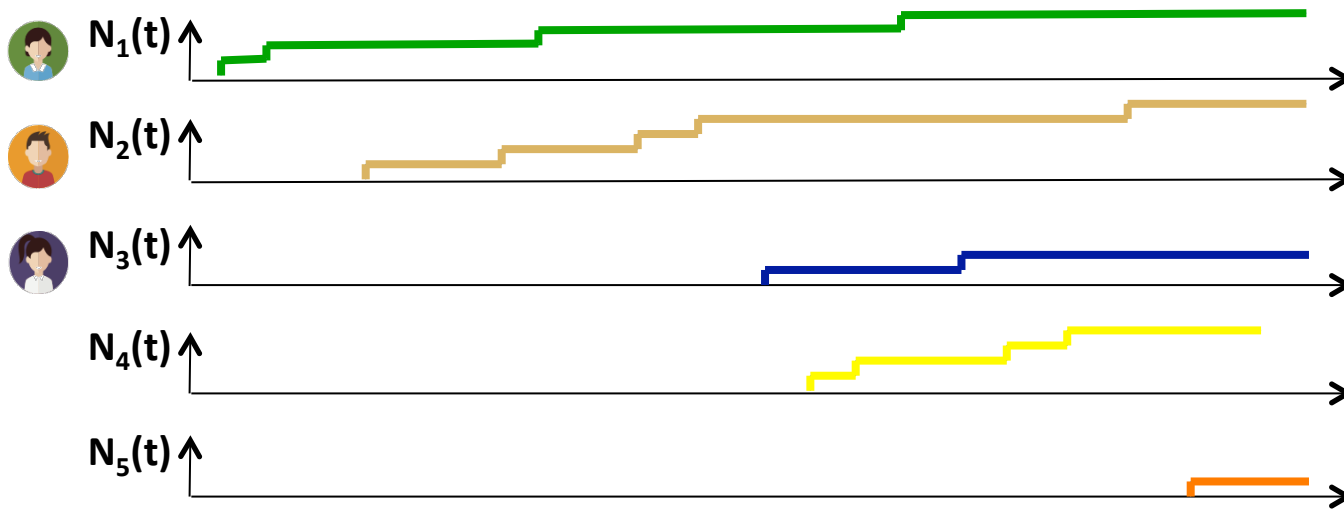
We represent messages using **nonterminating temporal point processes**:



**Recurrent event:**



# Events intensity



**Exogenous activity**

↓

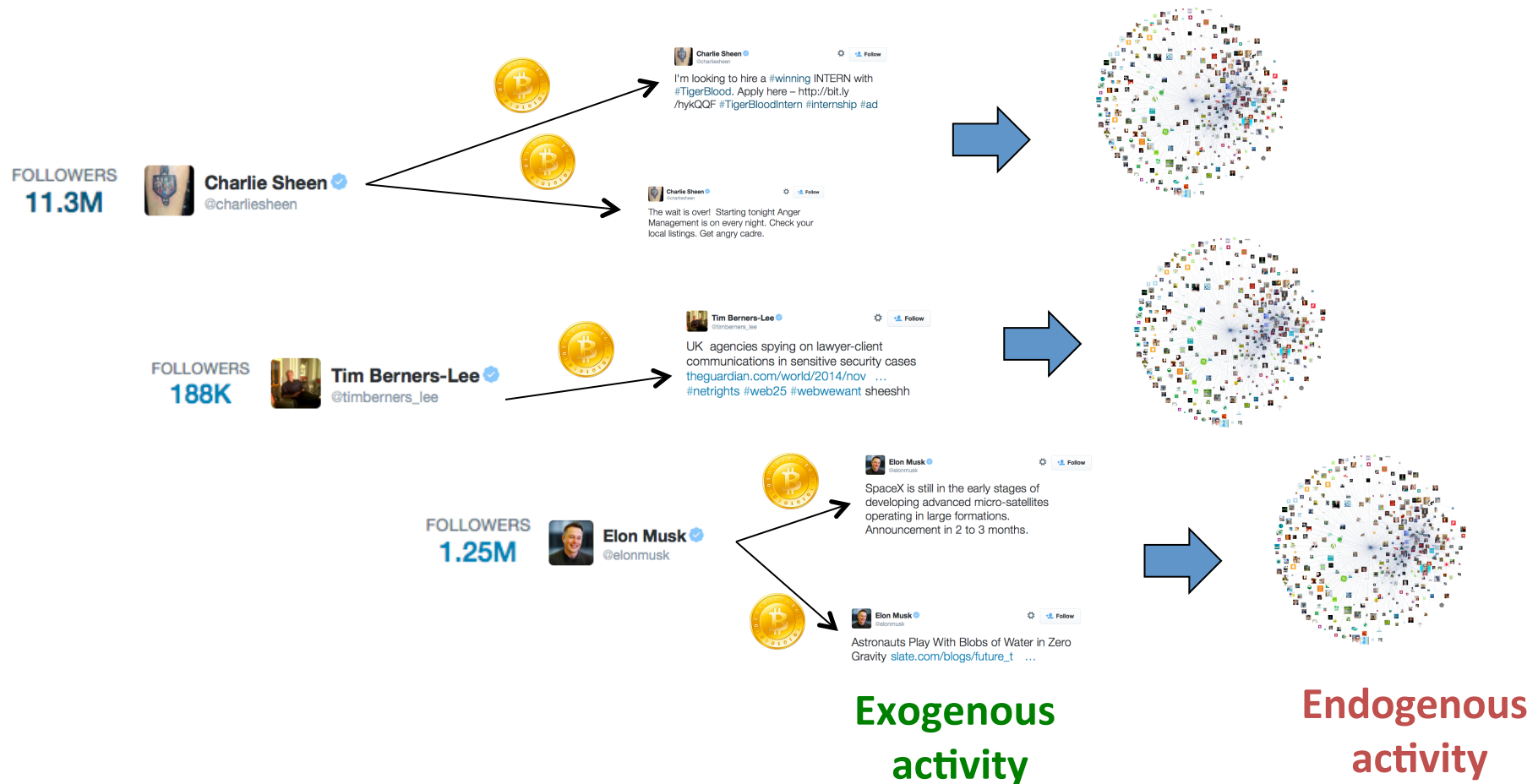
$$\underbrace{\lambda_u^*(t)}_{\text{User's intensity}} = \underbrace{\lambda^{(0)}(t)}_{\text{Messages on her own initiative}} + \sum_{i:t_i < t} a_{uu_i} g(t - t_i)$$

Influence from user  $u_i$  on user  $u$

Hawkes process <sub>7</sub>

# Activity shaping... how?

Incentivize  
a few users to produce a given level  
of overall users' activity





# Activity shaping... what is it?

Activity Shaping:

Find exogenous activity  $\lambda^{(0)}(t)$   
that results in a desired average  
overall activity at a given time:

$$\mu(t) := \mathbb{E}_{\mathcal{H}_{t-}} [\lambda(t)]$$



Average with respect to  
the history of  
events up to t!

# Exogenous intensity & average overall intensity

$$\boldsymbol{\lambda}^{(0)}(t) \quad \longrightarrow \quad \boldsymbol{\mu}(t) := \mathbb{E}_{\mathcal{H}_{t-}} [\boldsymbol{\lambda}(t)]$$

How do they relate?

Surprisingly...

**linearly:**

$$\boldsymbol{\mu}(t) = \boldsymbol{\Psi}(t) \star \boldsymbol{\lambda}^{(0)}(t)$$

Convolution

matrix that  
depends on

$g(t)$  and  $(a_{uu'})$

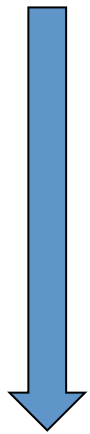
↑  
non negative  
kernel

↑  
influence  
matrix

# Exact Relation

If the memory  $g(t)$  is exponential:

$$\Psi(t) := \left( e^{(A - \omega I)t} + \omega(A - \omega I)^{-1} (e^{(A - \omega I)t} - I) \right)$$

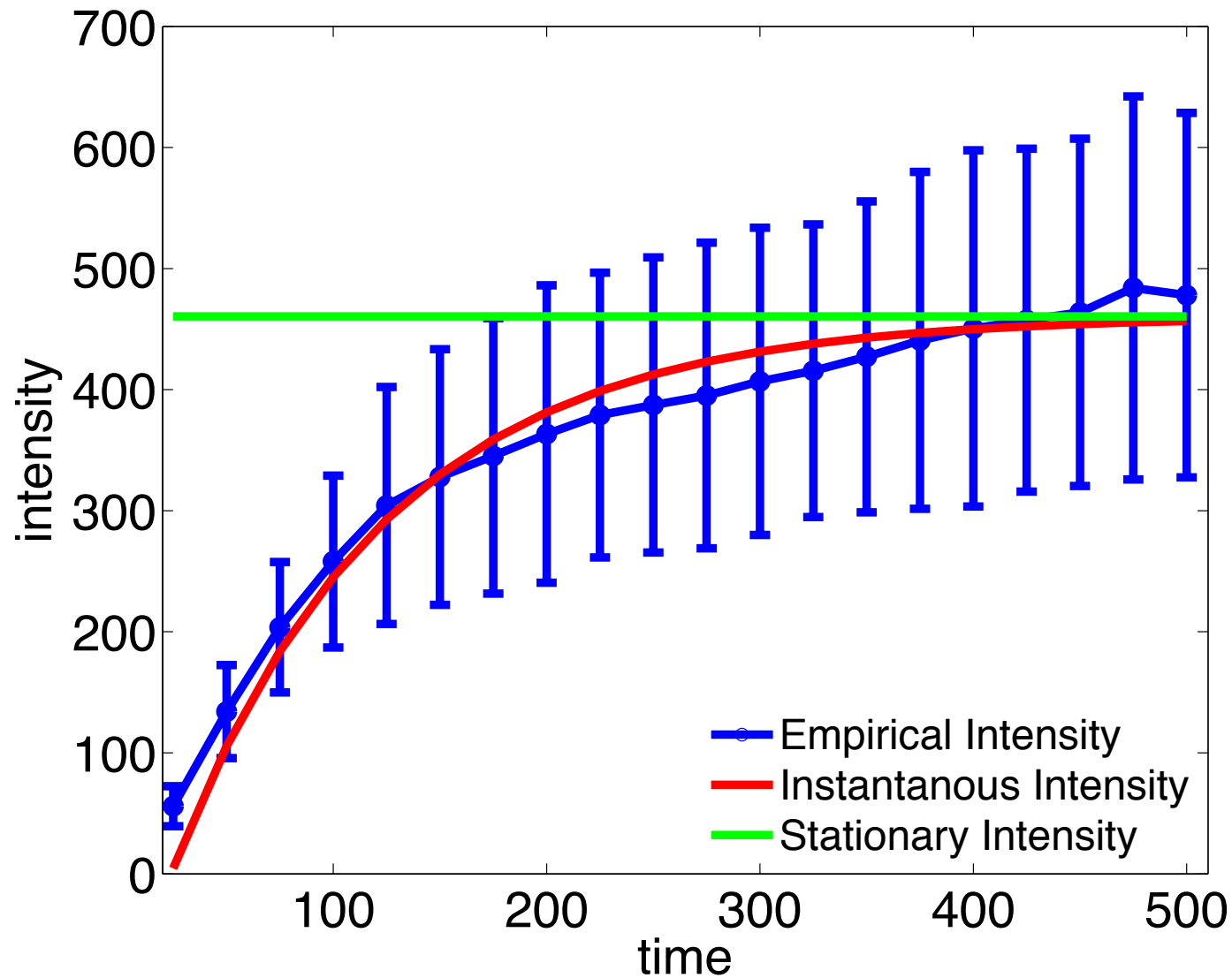


Matrix  
exponentials

$$\mu(t) = \Psi(t) \star \lambda^{(0)}(t) \quad \longrightarrow \quad \mu(t) = \Psi(t) \lambda^{(0)}$$

Corollary  
exogenous intensity  
is **constant**

# Does it really work in practice?



# Activity shaping optimization framework

Once we know that  $\mu(t) = \Psi(t)\lambda^{(0)}$

we can find  $\lambda^{(0)}$  to satisfy **many different goals**:

**ACTIVITY SHAPING**

We can solve this problem  
**efficiently for a large  
family of utilities!**

maximize  $\mu(t)$   
subject to

$\lambda^{(0)} \geq 0$



**Cost for incentivizing**

# Capped activity maximization (CAM)

If our goal is **maximizing the overall number of events** across a social network:

Max feasible  
activity per user



$$\begin{aligned} &\text{maximize}_{\boldsymbol{\mu}(t), \boldsymbol{\lambda}^{(0)}} && \sum_{u \in [m]} \min \{ \mu_u(t), \alpha_u \} \\ &\text{subject to} && \boldsymbol{\mu}(t) = \boldsymbol{\Psi}(t) \boldsymbol{\lambda}^{(0)}, \quad \mathbf{c}^\top \boldsymbol{\lambda}^{(0)} \leq C, \quad \boldsymbol{\lambda}^{(0)} \geq 0 \end{aligned}$$



**7 Ways to Increase Your Social Media Engagement**

# Minimax activity shaping (MMASH)

If our goal is make the user with the minimum activity as active as possible:

$$\text{maximize}_{\mu(t), \lambda^{(0)}} \min_u \mu_u(t)$$

$$\text{subject to} \quad \mu(t) = \Psi(t)\lambda^{(0)}, \quad \mathbf{c}^\top \lambda^{(0)} \leq C, \quad \lambda^{(0)} \geq 0$$

**BUSINESS  
INSIDER**

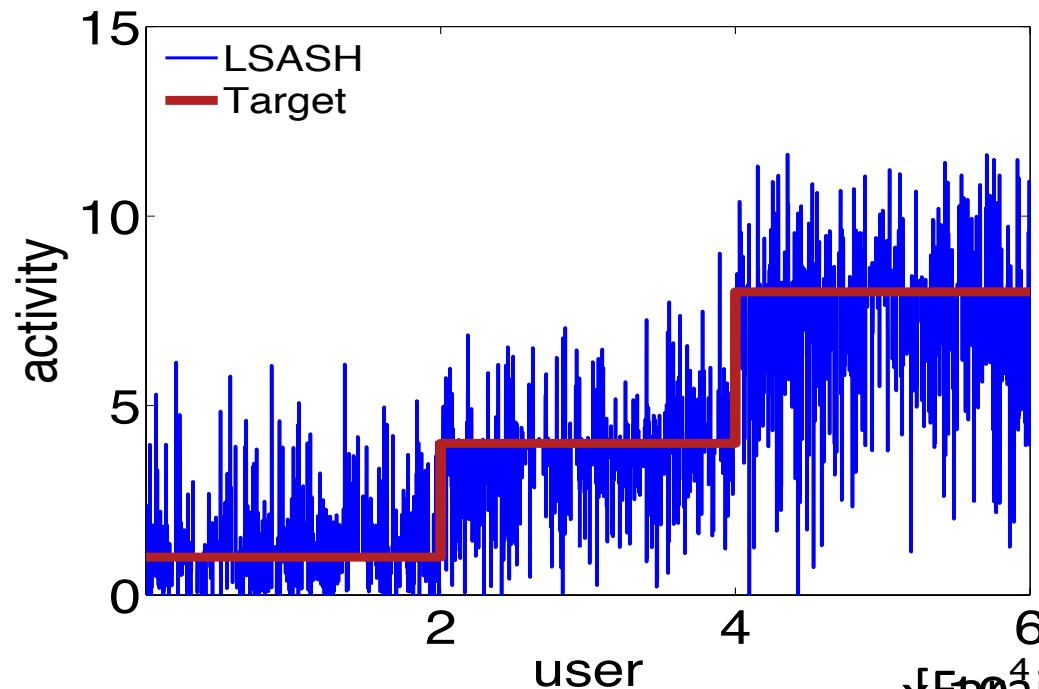
**1% Of Players Provide The Biggest Chunk Of Zynga's Revenue**

# Least-squares activity shaping (LSASH)

If our goal is to achieve a pre-specified level of activity for each user or group of users:

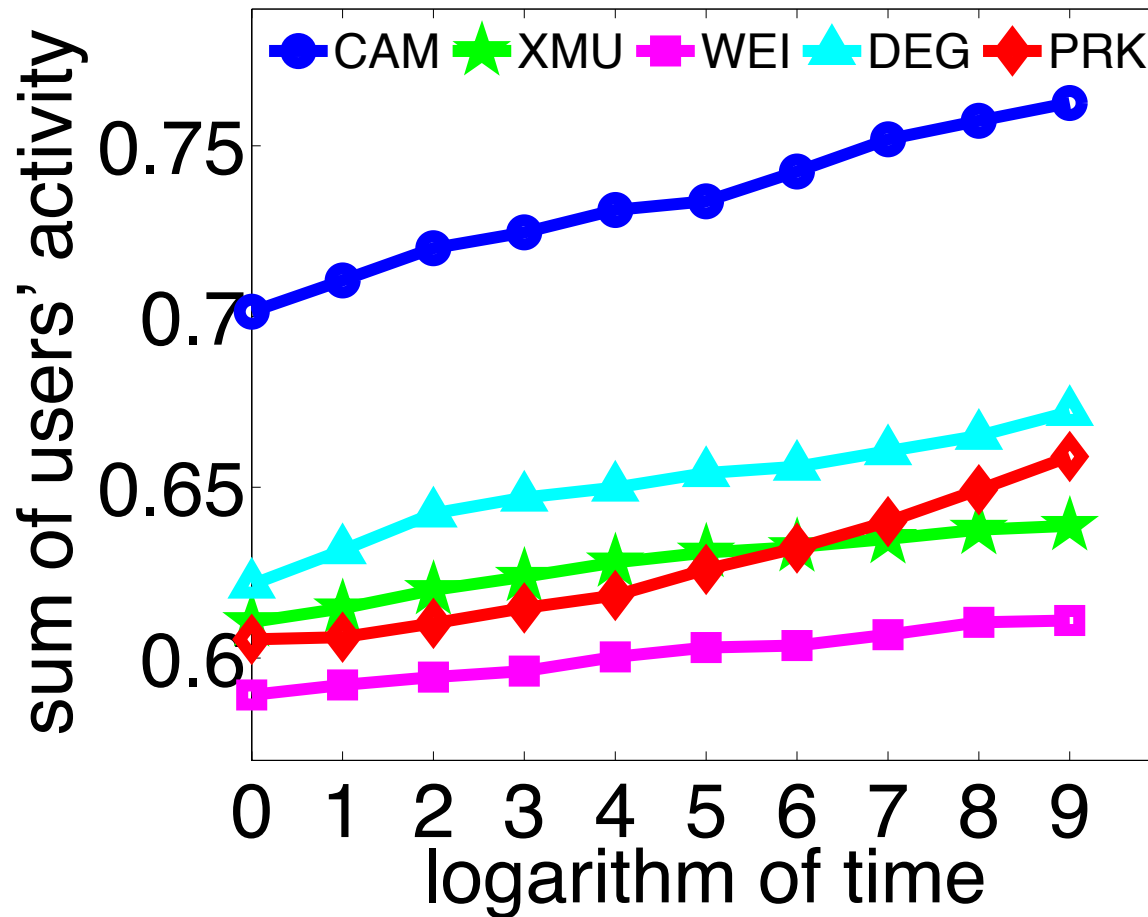
$$\text{maximize}_{\boldsymbol{\mu}(t), \boldsymbol{\lambda}^{(0)}} -\|\mathbf{B}\boldsymbol{\mu}(t) - \mathbf{v}\|_2^2$$

$$\text{subject to} \quad \boldsymbol{\mu}(t) = \boldsymbol{\Psi}(t)\boldsymbol{\lambda}^{(0)}, \quad \mathbf{c}^\top \boldsymbol{\lambda}^{(0)} \leq C, \quad \boldsymbol{\lambda}^{(0)} \geq 0$$





# Capped activity maximization: results



+10% more events  
than best heuristic



+34,000 more events per  
month than best heuristic for  
2,000 Twitter users

# Applications: Control

1. Activity shaping
- 2. When-to-post**

# Social media as a broadcasting platform

Everybody can build, reach and broadcast information to their own audience

twitter 



Broadcasted content

Audience reaction

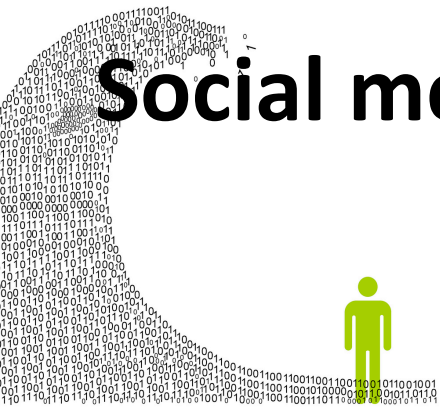


  
Instagram



# Attention is scarce

## Social media users follow many broadcasters



Instagram feed



Older posts



Twitter feed



Older posts



# What are the best times to post?

THE BLOG

THE HUFFINGTON POST

The Best Times to Post on Social Media

Tech.Mic

Here Are the Best Times to Post on Social Media So Your Picture

Can we design an algorithm that tell us when to post to achieve **high visibility**?

COMPANY  
s To Post On

HubSpot

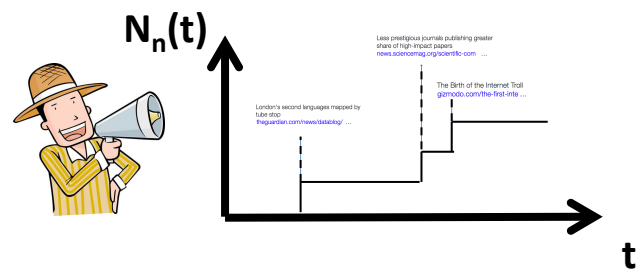
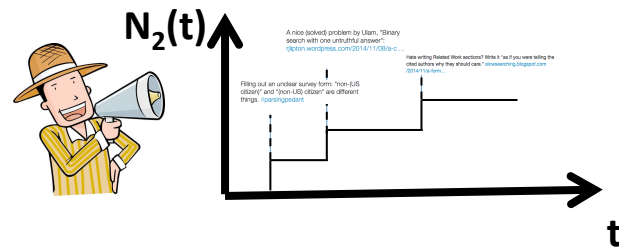
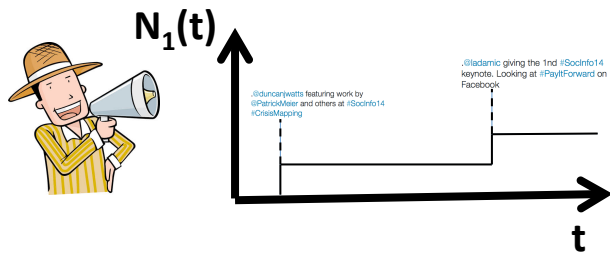
The Best Times to Post on Twitter, LinkedIn & Other Social Media Sites [Infographic]

Forbes

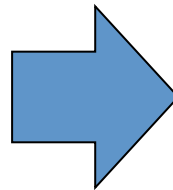
For Brands And PR: When Is The Best Time To Post On Social Media?

# Representation of broadcasters and feeds

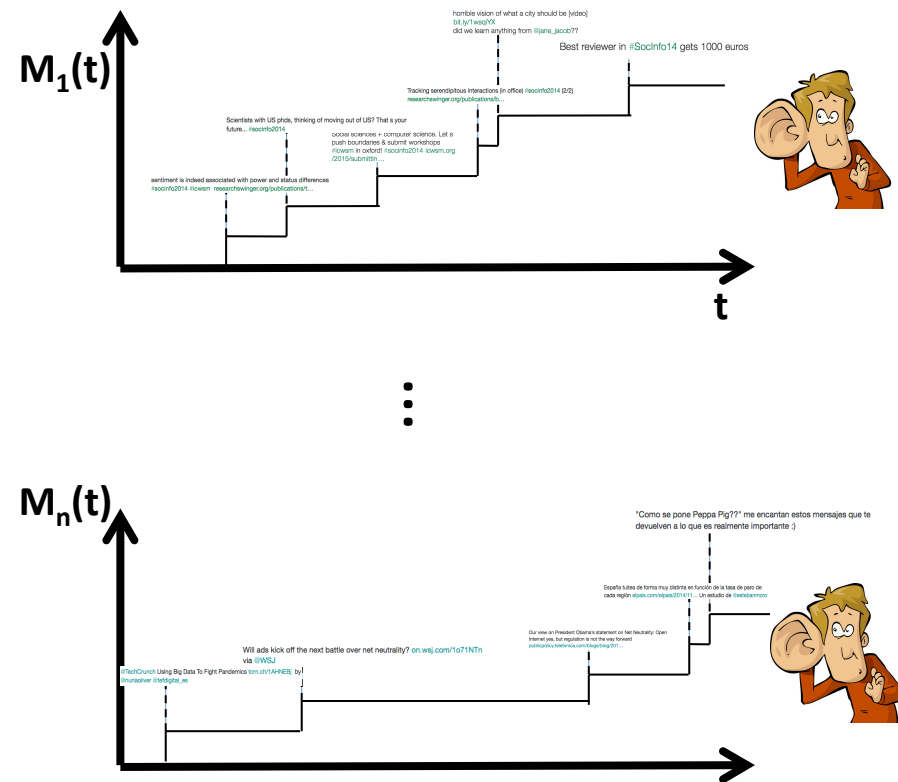
Broadcasters' posts as a counting process  $N(t)$



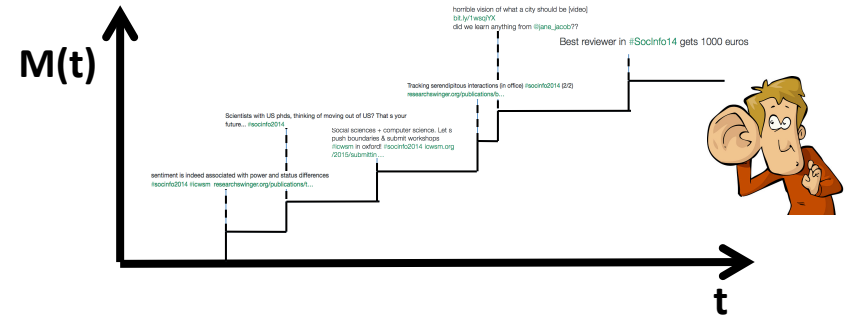
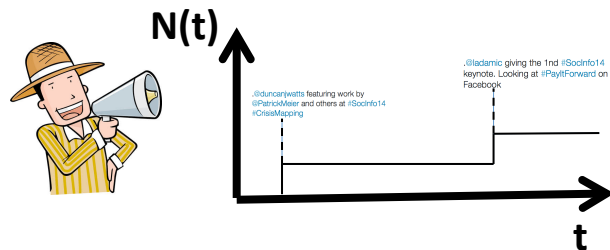
$$M(t) = A^T N(t)$$



Users' feeds as sum of counting processes  $M(t)$



# Broadcasting and feeds intensities



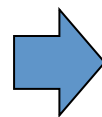
$$\mathbb{E}[dN(t)|\mathcal{H}(t)] = \underbrace{\mu(t)}_{\substack{\text{Broadcaster} \\ \text{intensity function} \\ \text{(tweets / hour)}}} dt$$

$\in \{0, 1\}$

$$\mathbb{E}[dM(t)|\mathcal{H}(t)] = \underbrace{\gamma(t)}_{\substack{\text{Feed intensity function} \\ \text{(tweets / hour)}}} dt$$

$\in \{0, 1\}$   $A^T \mu(t)$

Given a broadcaster  $i$  and her followers



$$M_{\setminus i}(t) = A^T N(t) - A_i N_i(t)$$

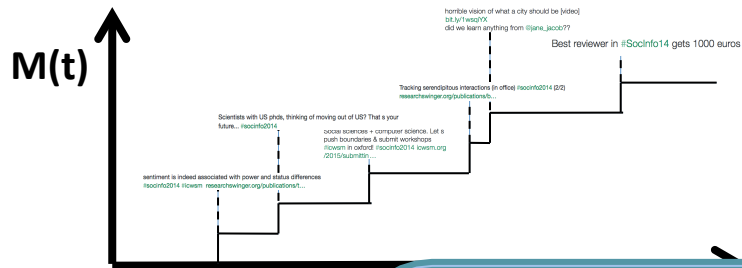
$$\gamma_{j \setminus i}(t) = \gamma_j(t) - \mu_i(t)$$

**Feed due to other broadcasters**

# Definition of visibility function

Visibility of broadcaster  $i$  at follower  $j$

Position of the highest ranked tweet by broadcaster  $i$  in follower  $j$ 's wall



$$r_{ij}(t) = 0$$

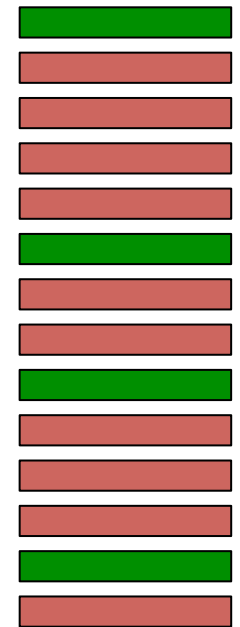
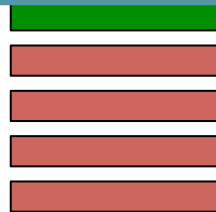
$$r_{ij}(t') = 4$$

$$r_{ij}(t'') = 0$$

In general, the **visibility** depends on the **feed ranking mechanism!**

Feed ranking

Ranked stories



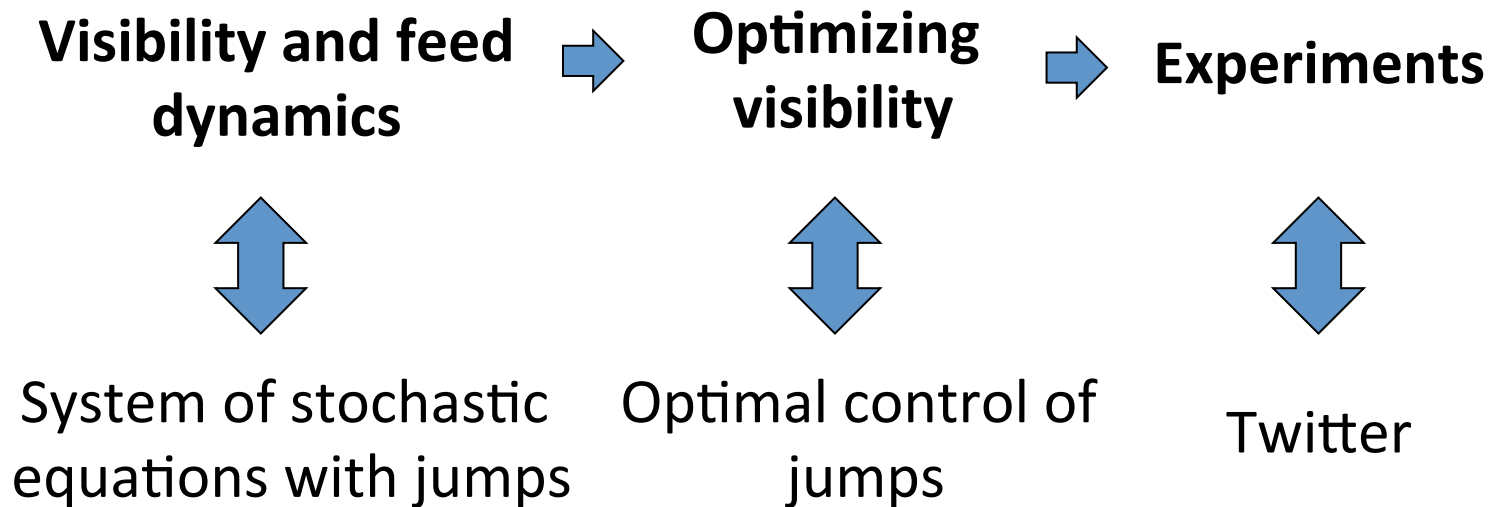
█ Post by broadcaster  $u$

█ Post by other broadcasters

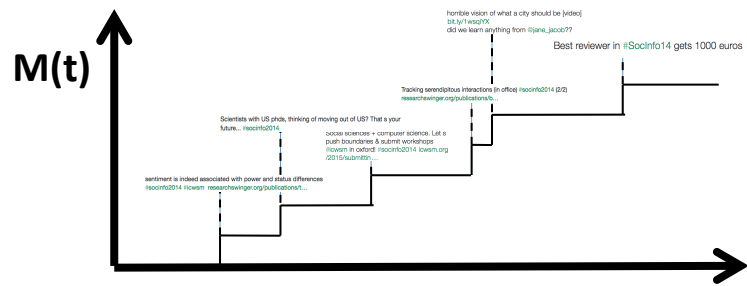


# Optimal control of temporal point processes

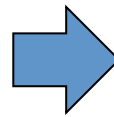
**Formulate the when-to-post problem as a novel stochastic optimal control problem**  
(of independent interest)



# Visibility dynamics in a FIFO feed (I)

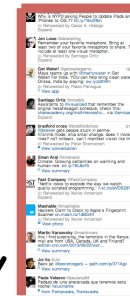


Reverse  
chronological order



New tweets

Older tweets



$$r_{ij}(t + dt) = \underbrace{(r_{ij}(t) + 1)}_{\text{Rank at } t+dt} dM_{j \setminus i}(t) \underbrace{(1 - dN_i(t))}_{\text{Other broadcasters post a story and broadcaster i does not post}} + \underbrace{0}_{\text{Broadcaster i posts a story and other broadcasters do not post}} + r_{ij}(t) \underbrace{(1 - dM_{j \setminus i}(t))}_{\text{Nobody posts a story}} (1 - dN_i(t))$$

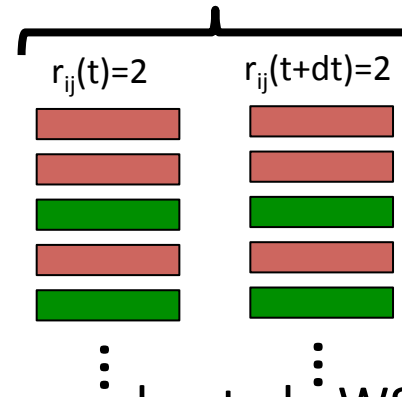
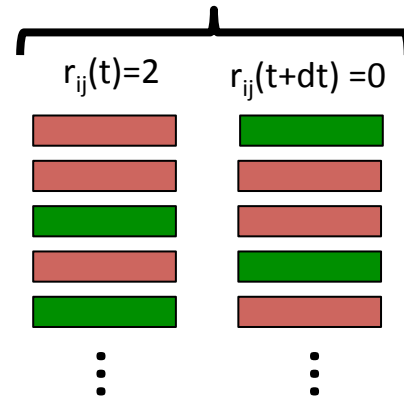
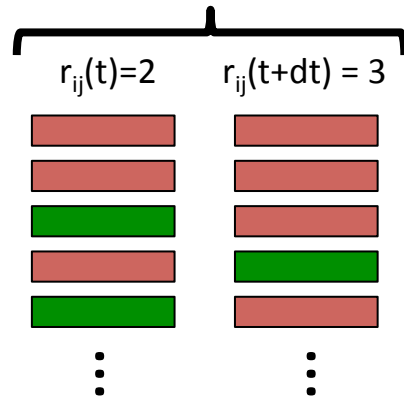
Rank at t+dt

Other broadcasters  
post a story and  
broadcaster i does  
not post

Broadcaster i  
posts a story and  
other broadcasters  
do not post

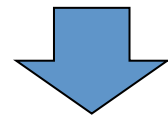
Nobody posts  
a story

Follower's wall



# Visibility dynamics in a FIFO feed (II)

$$r_{ij}(t + dt) = (r_{ij}(t) + 1)dM_{j \setminus i}(t)(1 - dN_i(t)) + 0 + r_{ij}(t)(1 - dM_{j \setminus i}(t))(1 - dN_i(t))$$



Zero-one law  $dN_i(t)dM_{j \setminus i}(t) = 0$

$$dr_{ij}(t) = -r_{ij}(t) dN_i(t) + dM_{j \setminus i}(t)$$



$$r_{ij}(t + dt) - r_{ij}(t)$$

Broadcaster  $i$   
posts a story

Other broadcasters  
posts a story

**Stochastic  
differential equation  
(SDE) with jumps**

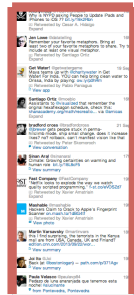
## OUR GOAL:

Optimize  $r_{ij}(t)$  over time, so that it is small, by controlling  $dN_i(t)$  through the intensity  $\mu_i(t)$

# Feed dynamics



$$N_i(t) \quad M_{j \setminus i}(t) = M(t) \rightarrow \gamma_{j \setminus i}(t) = \lambda(t)$$



We consider a  
**general intensity:**

(e.g. Hawkes,  
inhomogeneous Poisson)

$$\lambda^*(t) = \underbrace{\lambda_0(t)}_{\text{Deterministic arbitrary intensity}} + \underbrace{\alpha \int_0^t g(t-s) dN(s)}_{\text{Stochastic self-excitation}}$$



**Jump stochastic differential equation (SDE)**

$$\left\{ \begin{aligned} d\lambda^*(t) &= [\lambda'_0(t) + w\lambda_0(t) - w\lambda^*(t)] dt + \alpha dN_i(t) \end{aligned} \right.$$

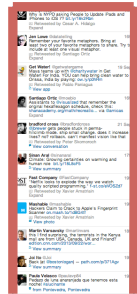
[Zarezade et al., WSDM 2017]

# The when-to-post problem

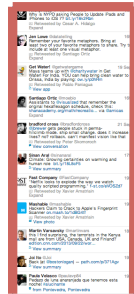


$$\mu_i(t) = u(t) \rightarrow N_i(t)$$

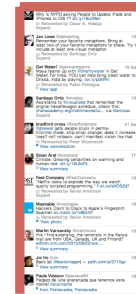
$$N_i(t) \quad M_{j \setminus i}(t)$$



$$N_i(t) \quad M_{j \setminus i}(t)$$



$$N_i(t) \quad M_{j \setminus i}(t)$$



$$dr_{ij}(t) = -r_{ij}(t) dN_i(t) + dM_{j \setminus i}(t)$$

$$dr_{ij}(t) = -r_{ij}(t) dN_i(t) + dM_{j \setminus i}(t)$$

$$dr_{ij}(t) = -r_{ij}(t) dN_i(t) + dM_{j \setminus i}(t)$$

Terminal penalty

Nondecreasing loss

Optimization problem

$$\text{minimize}_{u(t_0, t_f)} \mathbb{E}_{(N_i, M_{\setminus i})(t_0, t_f)} \left[ \phi(\mathbf{r}(t_f)) + \int_{t_0}^{t_f} \ell(\mathbf{r}(\tau), u(\tau)) d\tau \right]$$

$$\text{subject to } u(t) \geq 0 \quad \forall t \in (t_0, t_f],$$

Dynamics defined by Jump SDEs

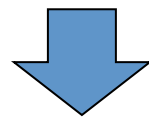
$$dr(t) = -r(t) dN(t) + dM(t)$$

$$d\lambda(t) = [\lambda'_0(t) + w\lambda_0(t) - w\lambda(t)] dt + \alpha dM(t)$$

# Bellman's Principle of Optimality

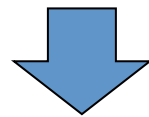
**Lemma. The optimal cost-to-go satisfies Bellman's Principle of Optimality**

$$J(r(t), \lambda(t), t) = \min_{u(t, t+dt)} \mathbb{E} [J(r(t+dt), \lambda(t+dt), t+dt)] + \ell(r(t), u(t)) dt$$



$$J(r(t+dt), \lambda(t+dt), t+dt) = J(r(t), \lambda(t), t) + dJ(r(t), \lambda(t), t)$$

$$0 = \min_{u(t, t+dt)} \mathbb{E} [dJ(r(t), \lambda(t), t)] + \ell(r(t), u(t)) dt$$



$$\begin{aligned} dr(t) &= -r(t) dN(t) + dM(t) \\ d\lambda(t) &= [\lambda'_0(t) + w\lambda_0(t) - w\lambda(t)] dt + \alpha dM(t) \end{aligned}$$

**Hamilton-Jacobi-Bellman (HJB)  
equation**



**Partial differential  
equation in J  
(with respect to  $r$ ,  $\lambda$  and  $t$ )**

[Zaregade et al., WSDM 2017]

# Solving the HJB equation

Consider a quadratic loss

$$\ell(r(t), u(t)) = \frac{1}{2} \underset{\uparrow}{s(t)} r^2(t) + \frac{1}{2} \underset{\uparrow}{q} u^2(t)$$

Favors some periods of times

(e.g., times in which the follower is online)

Trade-offs visibility and number

of broadcasted posts

We propose  $J(r(t), \lambda(t), t)$  and then show that the optimal intensity is:

$$\begin{aligned} u^*(t) &= q^{-1} [J(r(t), \lambda(t), t) - J(0, \lambda(t), t)] \\ &= \sqrt{s(t)/q} r(t) \end{aligned}$$

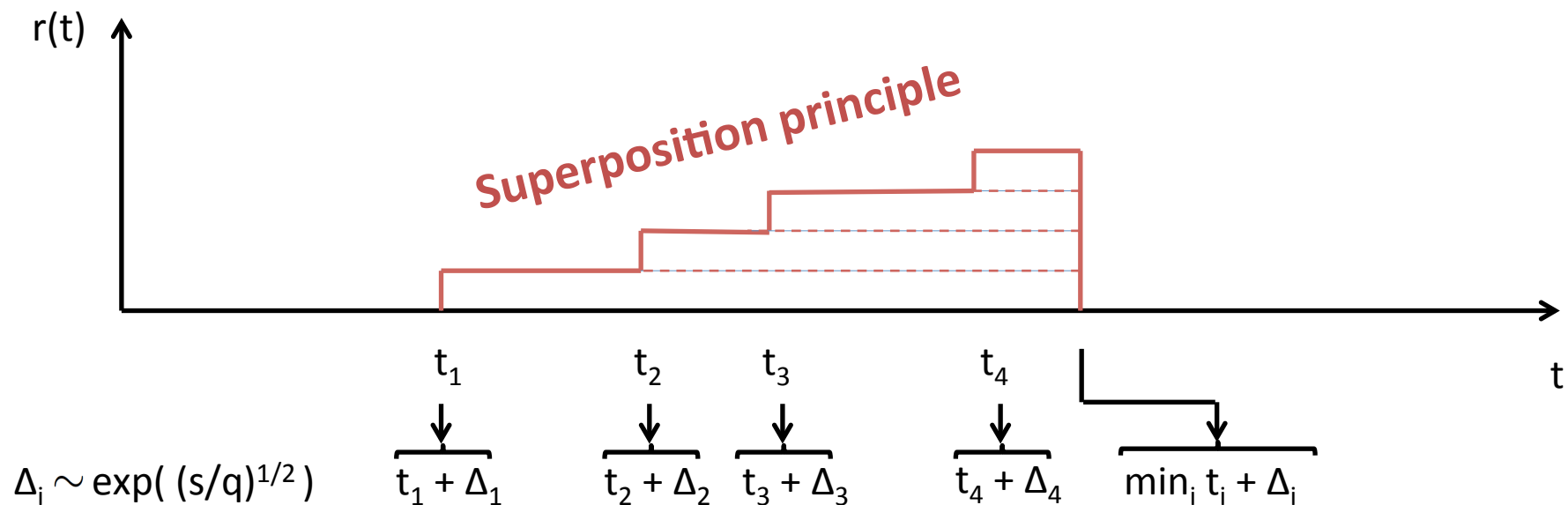
**It only depends on the current visibility!**



# The RedQueen algorithm

Consider  $s(t) = s \longrightarrow u^*(t) = (s/q)^{1/2} r(t)$

How do we sample the next time?



It only requires sampling  $M(t_f)$  times!





# The RedQueen algorithm

RedQueen can be implemented in a few lines of code!

---

**Algorithm 1:** REDQUEEN for fixed  $s$ ,  $q$  and one follower.

---

**Input:** Parameters  $q$  and  $s$

**Output:** Returns time for the next post

$t \leftarrow \infty$ ;  $\tau \leftarrow \text{othersNextPost}()$

**while**  $\tau < t$  **do**

$\Delta \sim \exp(\sqrt{s/q})$

$t \leftarrow \min(t, \tau + \Delta)$

$\tau \leftarrow \text{othersNextPost}()$

**end**

**return**  $t$

---

# When-to-post for multiple followers

Consider  $n$  followers and a quadratic loss:

$$\ell(\mathbf{r}(t), u(t), t) = \sum_{i=1}^n \frac{1}{2} s_i(t) r_i^2(t) + \frac{1}{2} q u^2(t)$$

We can easily adapt the efficient sampling algorithm to multiple followers!

$i=1$   
It only depends on the current visibilities!

# Novelty in the problem formulation

The problem formulation is unique in two key technical aspects:

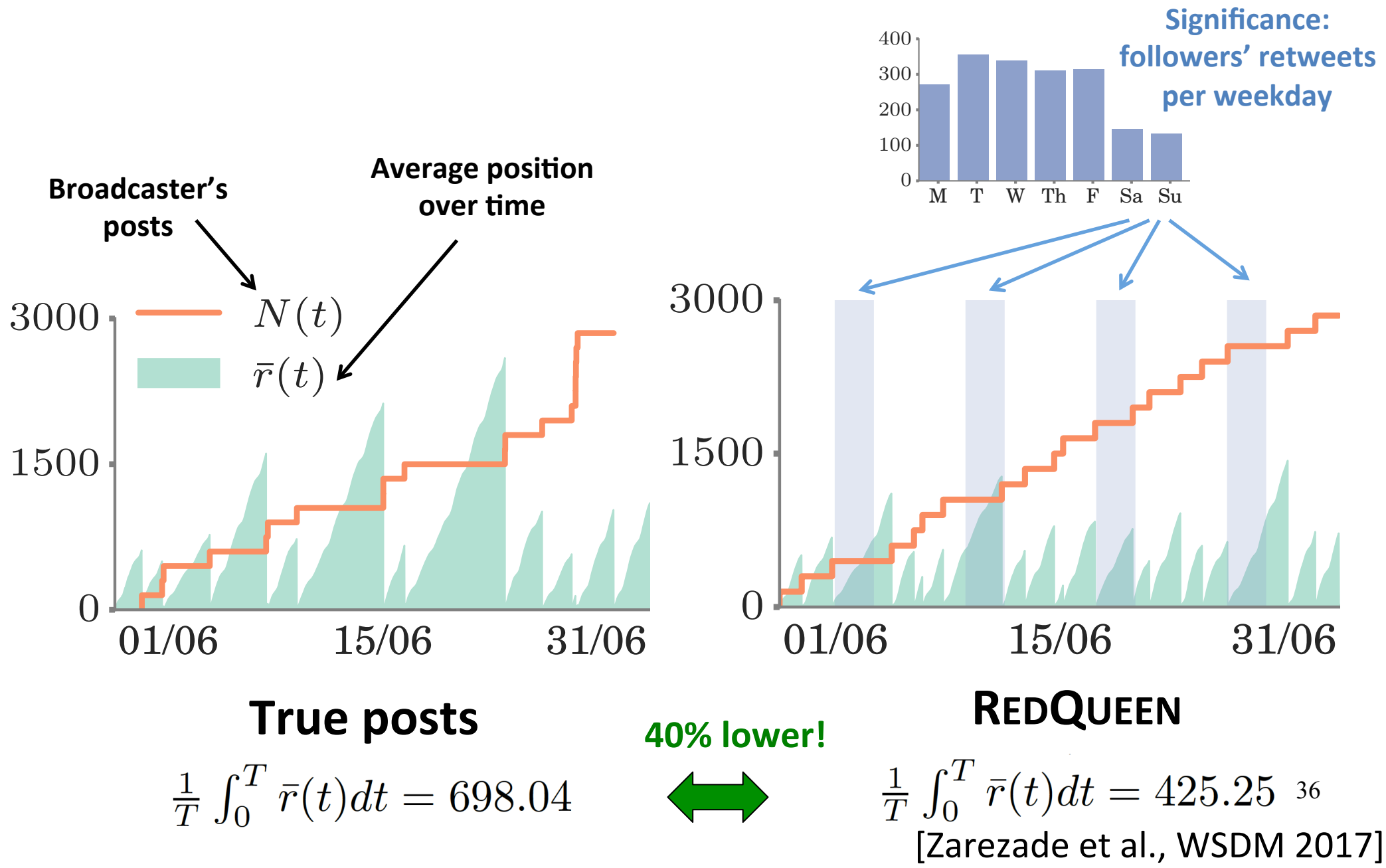
## **I. The control signal is a conditional intensity**

Previous work: time-varying real vector

## **II. The jumps are doubly stochastic**

Previous work: memory-less jumps

# Case study: one broadcaster



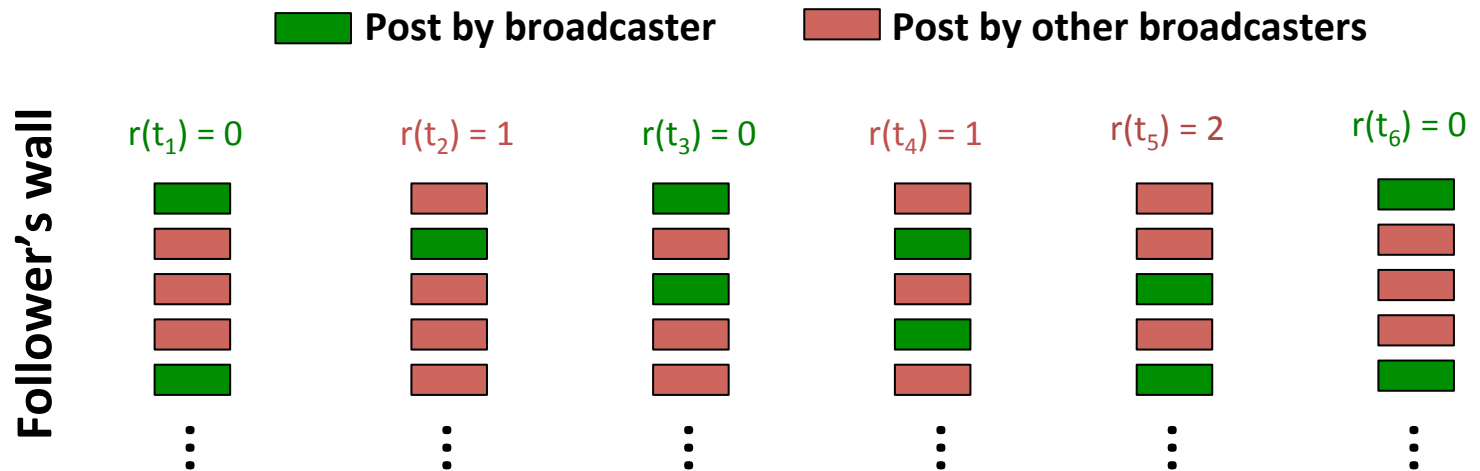
# Evaluation metrics

Position over time

$$\int_0^T r(t) dt$$

Time at the top

$$\int_0^T \mathbb{I}(r(t) < 1) dt$$



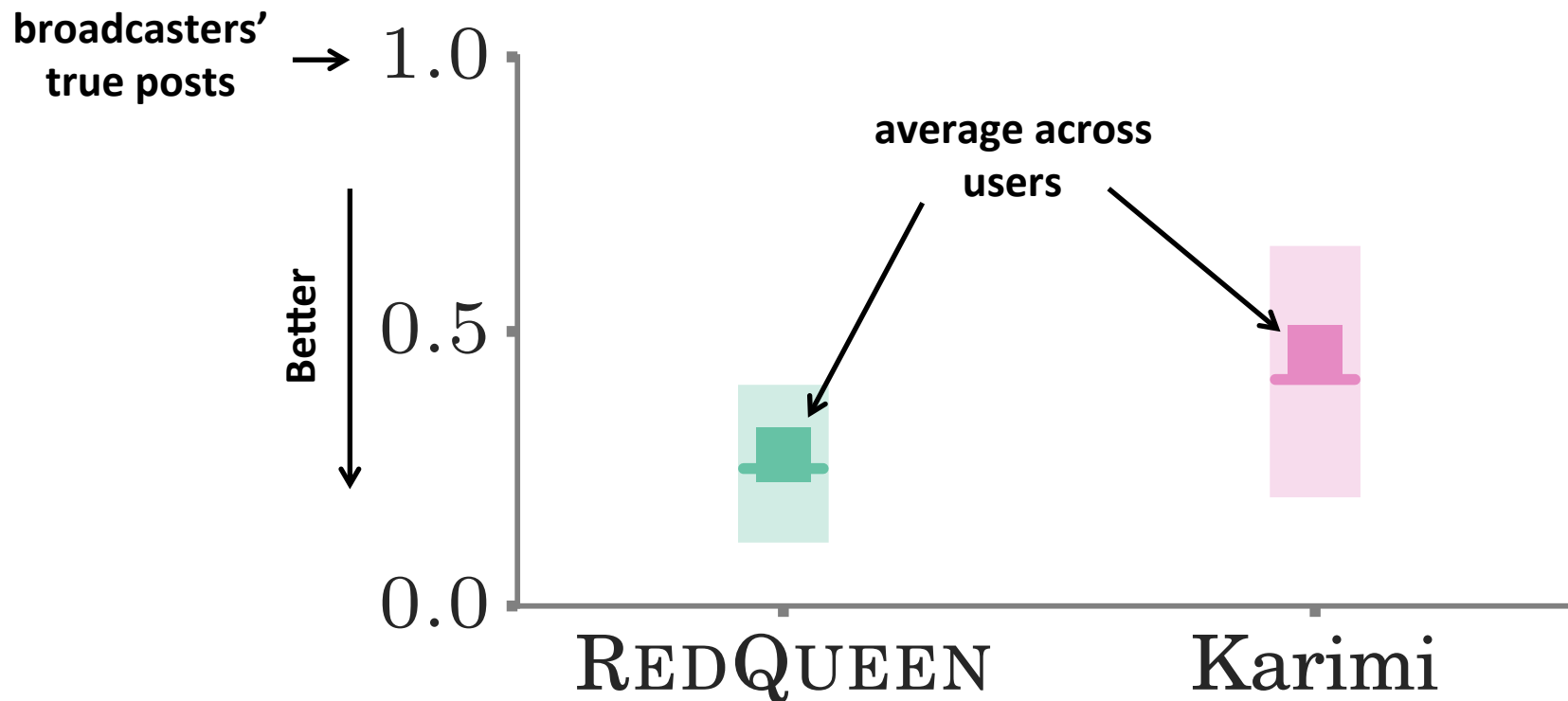
Position over time =

$$0x(t_2 - t_1) + 1x(t_3 - t_2) + 0x(t_4 - t_3) + 1x(t_5 - t_4) + 2x(t_6 - t_5)$$

Time at the top =

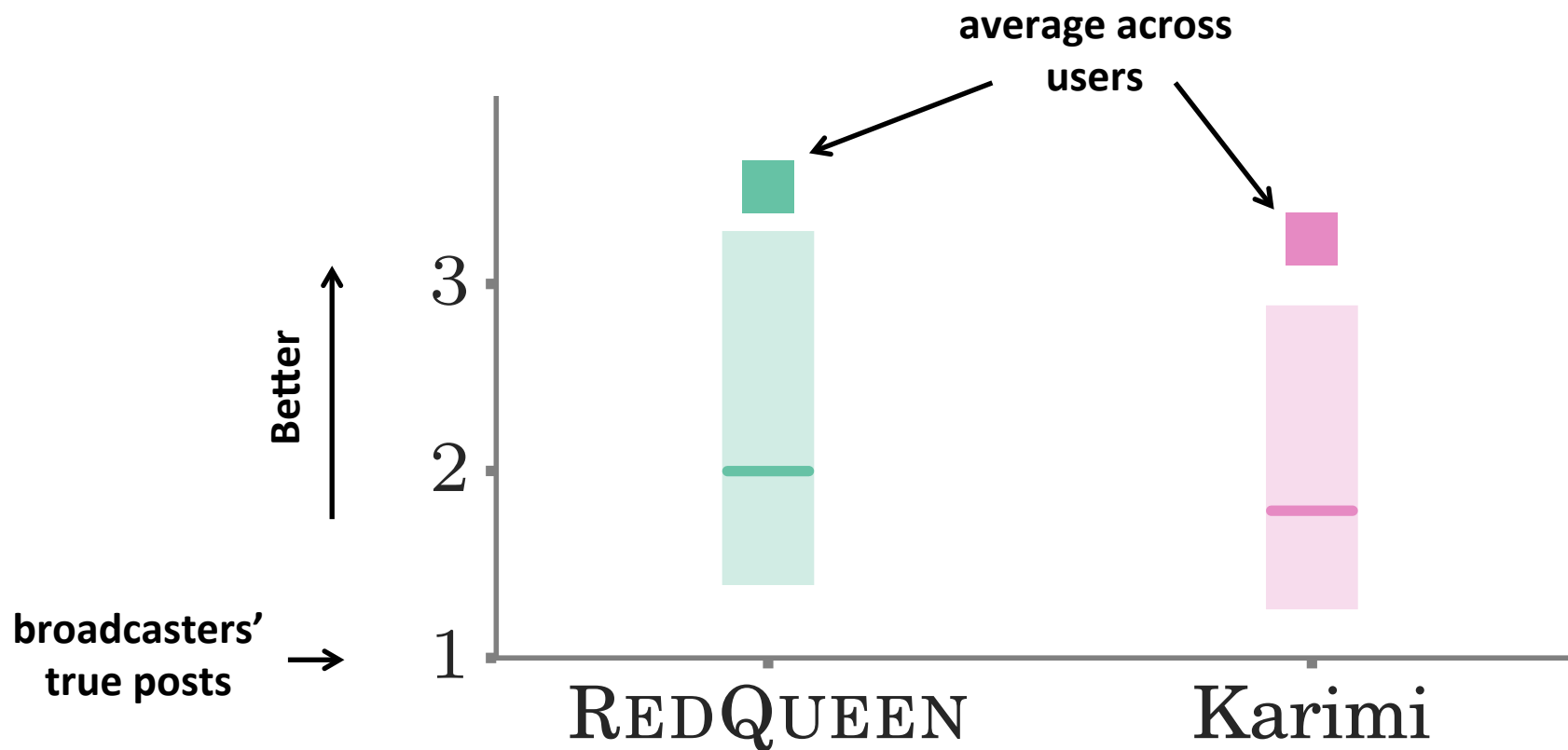
$$(t_2 - t_1) + 0 + (t_4 - t_3) + 0 + 0$$

# Position over time



It achieves (i) **0.28x lower average position, in average, than the broadcasters' true posts** and (ii) **lower average position for 100% of the users.**

# Time at the top



It achieves (i) **3.5x higher time at the top, in average, than the broadcasters' true posts** and (ii) **higher time at the top for 99.1% of the users.**

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## **APPLICATIONS: MODELS**

1. Information propagation
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## **APPLICATIONS: CONTROL**

1. Activity shaping
2. When-to-post